Lecture 1

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1 Financial Assets

We use the term financial asset to mean a financial instrument which can be bought and sold, like a stock, bond, gold, or a currency. First we describe some basic assets that are available in financial markets. Second we describe some derivative assets, like futures and options, whose price is derived from the price of the basic assets.

1.1 Stocks

Stocks are securities representing equity ownership in a corporation. The owners of a stock have a limited liability. This implies that the price of a stock is always nonnegative, so that the price S_t of a stock at time t satisfies

$$0 \leq S_t < \infty.$$

1.2 Stock Indeces

1.2.1 Definition of a Stock Index

Stock indices are usually averages of stock prices, weighted by market capitalization: the value of a stock index of N stocks at time t is calculated by formula

$$I_t = C \sum_{i=1}^{N} n_i S_t^i, \tag{1}$$

where C is a constant, n_i is the number of shares of stock i, and S_t^i is a suitably adjusted price of stock i at time t. The definition of the stock index depends on three parameters: the constant C, the numbers n_i, and the numbers S_t^i.

1. The constant C can be chosen, for example, to make the value of the index equal to 100 at a given past day. When the constitution of the index is changed, then the constant C is changed, to keep the index equal to 100 at the chosen day.

2. The value of n_i can be the total number of shares of stock i, but it can also be the number of freely floating stocks. Float market capitalization excludes stocks which are not freely floating (cannot be bought in the open market).

3. The value S_t^i is calculated differently depending on whether the index is a *price return index* or a *total return index*. Price return indeces are calculated without regard to cash dividends but total return indeces are calculated by reinvesting cash dividends. The *adjusted close* of a stock is the daily closing price of a stock which is adjusted to cash dividends, stock dividends, stock splits, and also to more complex corporate actions, such as rights offerings. The calculation of the adjusted closing price is often made by data providers.

1.2.2 Examples of Stock Indeces

U.S. Indeces We mention Dow Jones Industrial Average, S&P 500, and Nasdaq-100 stock indeces.

- Dow Jones Industrial average is an exception of the rule (1), because in its calculation the components are not weighted by the number of shares.

- S&P 500 was created at 4 March 1957. It was calculated back until 1928 and the basis value was taken to be 10 from 1941 until 1943.

  The S&P 500 is a price return index, but there exists also total return versions (dividends are invested back) and net total return versions (dividends minus taxes are invested back) of the S&P 500 index.

  The S&P 500 is a market value weighted index: prices of stocks are weighted according to the market capitalizations of the companies. Since 2005 the index is float weighted, so that the market capitalization is calculated using only stocks that are available for public trading.

- NASDAQ-100 is calculated since 31 Januar 1985. The basis value was at that day 250.

  NASDAQ-100 is a price index, so that the dividends are not included in the value of the index.

NASDAQ-100 is a different index than Nasdaq Composite, which is based on 3000 companies. NASDAQ-100 is calculated using the 100 largest companies in Nasdaq composite. NASDAQ-100 is a market value weighted index, but the influence of the largest companies is capped (the weight of any single company is not allowed to be larger than 24%).

European Indeces We mention DAX 30 stock index.

- DAX 30 (Deutscher AktienindeX) was created at 1. July 1988. The basis value is 1000 at 31 December 1987.

  DAX 30 is a performance index (dividends are reinvested in calculating the value of the index).

  DAX 30 stock index is a market value weighted index of 30 largest German companies. Market value is calculated using only free floating stocks (stocks that are not owned by an owner which has more than 5% of stocks). Largeness is measured by taking into account both the free floating market value and the transaction volume (total value of the stocks that are exchanged in a given time period). The weight of any single company is not allowed to be larger than 10%.

1.3 Fixed Income Instruments

Fixed income research studies how much one should pay today, in order to receive a cash payment at a future day. One unit of currency today is better than one unit of currency tomorrow, and fixed income research quantifies this phenomenon.

1.3.1 Zero-Coupon Bonds

A zero-coupon bond or pure discount bond is a certificate which gives the owner a nominal amount P at the future maturity time T.

1.3.2 Coupon Bearing Bonds

Most bonds make regular payments (coupons) before the final payment at maturity. A coupon bond can be defined as a series of payments $P_1, \ldots, P_N$ at times $T_1, \ldots, T_N$. The terminal payment contains the principal and the final coupon payment.

For example, a five year 4% semi-annual coupon bond with 1000$ face value makes ten 20$ payments every six months and the final payment of

1000\$. Thus $P_i = 20\$$ for $i = 1, \ldots, N-1$ and the last payment is $P_N = 1020\$$, where $N = 10$.

A zero-coupon bond is a more basic concept than a coupon bond, because a coupon bond can be defined as a portfolio of zero-coupon bonds. Let B_t be a coupon bond which starts at t_0 ans makes payments $P_1, \ldots, P_N$ at times $T_1 < \cdots < T_N$, where $T_1 > t_0$. It holds that

$$B_t = \sum_{i=1}^{N} P_i P(t_0, T_i),$$

where $P(t_0, T_i)$ is the zero coupon bond starting at t_0 with maturity T_i.

2 Derivatives

2.1 Forwards and Futures

2.1.1 Forward

A forward is a contract written at time t_0, with a commitment to accept delivery of (or to deliver) the specified number of units of the underlying asset at a future date T, at forward price F_{t_0}. The current price of the underlying is called the spot price. At time t_0 nothing changes hands, all exchanges will take place at time T. A long position is a commitment to accept the delivery at time T. A short position is a commitment to deliver the contracted amount.

2.1.2 Futures contract

A futures contract can be considered as a special case of a forward contract. An instrument is called a futures contract if the trading is done in a futures exchange, where the forward commitment is made through a homogenized contract so that the size of the underlying asset, the quality of the underlying asset, and the expiration date are preset. In addition, futures exchanges require a daily mark-to-market of the positions.

A futures exchange acts as an intermediary between the participants of a futures contract. The existence of the intermediary minimizes the risk of the default of the participants of the contract. When a participant enters a futures contract the exchange requires to put up an initial amount of liquid assets into the margin account. Marking to market means that the daily futures price is settled daily so that the exchange will draw money out of one party's margin account and put it into the other's so that the daily loss

or profit is taken into account. If the margin account goes below a certain value, then a margin call is made and the account owner must add money to the margin account. In contrast to futures contracts, forward contracts may not require any marking to market until the expiration day.

A futures contract can be settled with cash or with the delivery of the underlying. For example, if the underlying of the futures contract is a stock index, then the futures contract is usually settled with cash. A futures contract can be closed before the expiration day by entering the opposite direction futures contract.

2.2 Options

2.2.1 Calls and Puts

The buyer of a call option receives the right to buy the underlying instrument and the buyer of a put option receives the right to sell the underlying instrument.

An European call option gives the right to buy an asset at the given expiration time T at the given strike price K. An European put option gives the right to sell an asset at the given expiration time T at the given strike price K. Let us denote with C_t the price of an European call option at time t and with S_t the price of the asset. The value C_T of the European call option at the expiration time T is equal to

$$C_T = \max\{S_T - K, 0\}.$$

Let us denote with P_t the price of a put option at time t. The value of the European put option at the expiaration time T is equal to

$$P_T = \max\{K - S_T, 0\}.$$

There exists three basic modes concerning the right to exercise the option: European, American, and Bermudan. American call and put options can be exercised at any time before the expiration date. Thus an American option is more expensive than the corresponding European option. A Bermudean option can be exercised at some times or time periods before the expiration. When we use the term "option" without a further qualification, then we refer to an European option.

The following terminology is used to describe options.

- A call option is out of the money if $S_t < K$. A call option is at the money if $S_t = K$. A call option is in the money if $S_t > K$. A call option is deep out of the money (deep in the money) if $S_t \ll K$ ($S_t \gg K$).

- Before the expiration time T we have for a call option that

$$C_t > (S_t - K)_+.$$

The difference $C_t - (S_t - K)_+$ is called the time value of the option. The value $(S_t - K)_+$ is called the intrinsic value. Thus,

$$C_t = \text{time value} + \text{intrinsic value}.$$

- Moneyness is defined by K/S_t or by $Ke^{-r(T-t)}/S_t$, where $T - t$ is the time to expiration in fractions of year and r is the annualized short term interest rate.

2.2.2 Uses of Options

Options can serve many different purposes. We mention the following uses of options.

1. Options can be used to create a large number of different payoffs. For example, buying a call and a put with the same strike price and the same expiration creates a straddle position which profits from large positive or negative movements of the underlying.

2. Options can provide insurance. This is an example of creating a payoff with a desirable property: with options we can create a payoff which cuts the losses that could occur without using of the options. Buying a put option gives an insurance in the case one has to sell in a future time an asset one possesses. Buying a call option gives an insurance to the case when one has to buy in a future time an asset one does not possess. Examples of providing insurance with options include the following:

   - Buying a put option on a stock gives an insurance policy for an investor. If an investor owns a stock, buying a put option will cut the future possible losses.
   - Buying a put option on an exchange rate gives an insurance policy for a company receiving payments on a foreign currency in future.

3. A call option can be used to give a compensation to managers, since the payoff of a call option is positive only when the stock price is larger than the strike price.

4. Options allow the use of leveraging. The option trading requires a small initial capital as compared to the stock trading.[1]

2.2.3 Exotic options

We say that an option is exotic if it is not an European or an American call or put option. For example, Bermudean options are called exotic options. Bermudean options are such options that can be excercised during some time periods before the expiration time, whereas European options can be exercised only at the expiration date and American options can be exercised at any time before the expiration. Other exotic options include Asian options and Barrier options:

1. The value of an Asian call option at the expiration is

$$C_T = \max\{0, M_t - K\},$$

where

$$M_T = \frac{1}{N}\sum_{i=1}^{N} S_{t_i}$$

with $t_1 < \cdots < t_N = T$ being a collection of predetermined time points. Asian options are more resistant to manipulation than European options. The value of an European option at the expiration depends on the value of the underlying asset at one time point (the expiration date), whereas the value of an Asian option depends on the values of the underlying asset at several time points.

2. Barrier options disappear if the underlying either exceeds, or goes under the barrier. Alternatively, a barrier option could have value only if it has exceeded, or went under the barrier. Knock-in options come into existence if some barrier is hit and knock-out options cease to exist if some barrier is hit. One speaks of up-and-out, down-and-out, up-and-in, down-and-in options. We have

vanilla call, strike K
= knock-in, K-call, barrier H + knock-out, K-call, barrier H.

Barrier options are cheaper than the corresponding European options, which motivates their use.

[1]Suppose that the stock price is $S_{t_0} = 100$, the strike price is $K = 105$, and the call price is $C_{t_0} = 5$. If the stock price rises to $S_T = 110$ at the expiration time of the call option, then the owner of the stock has the return of 10% but the owner of the call option has the return of $(110 - 105)/5 = 100\%$.