Stylized Facts

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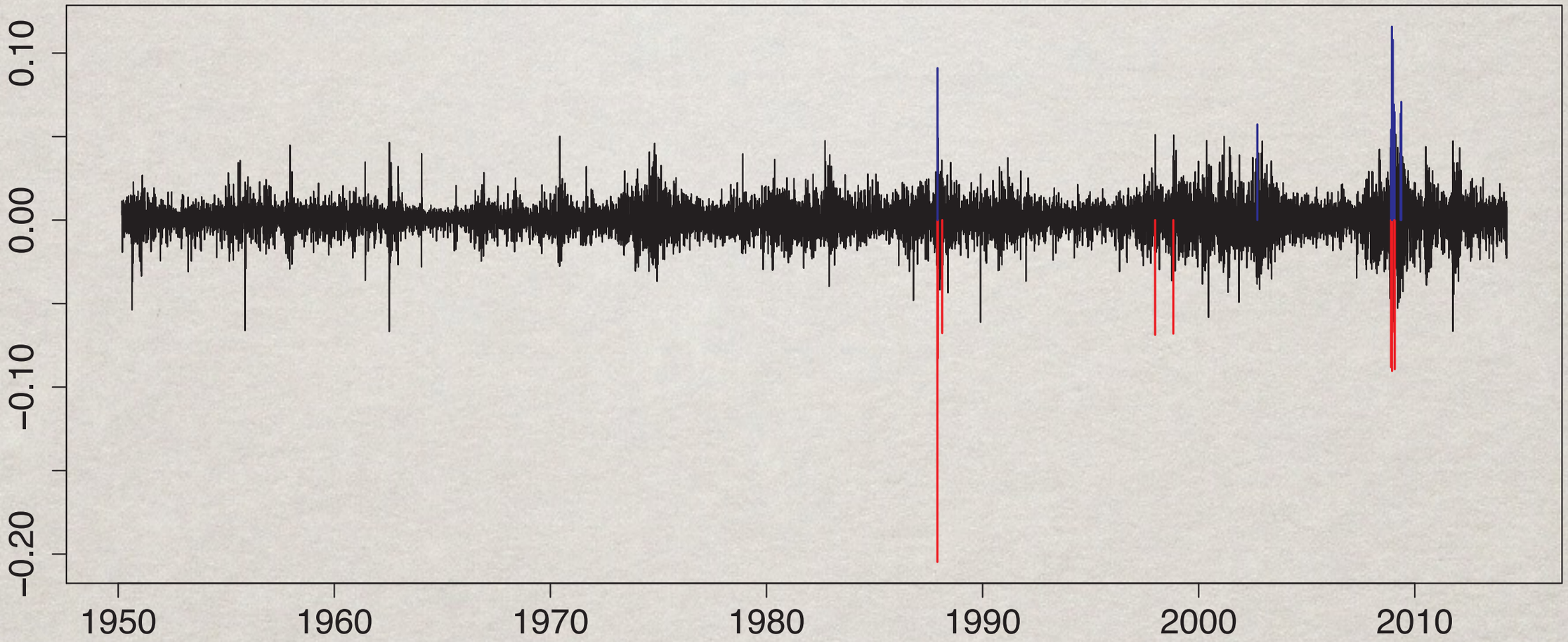
S&P 500 RETURNS



Simulated returns

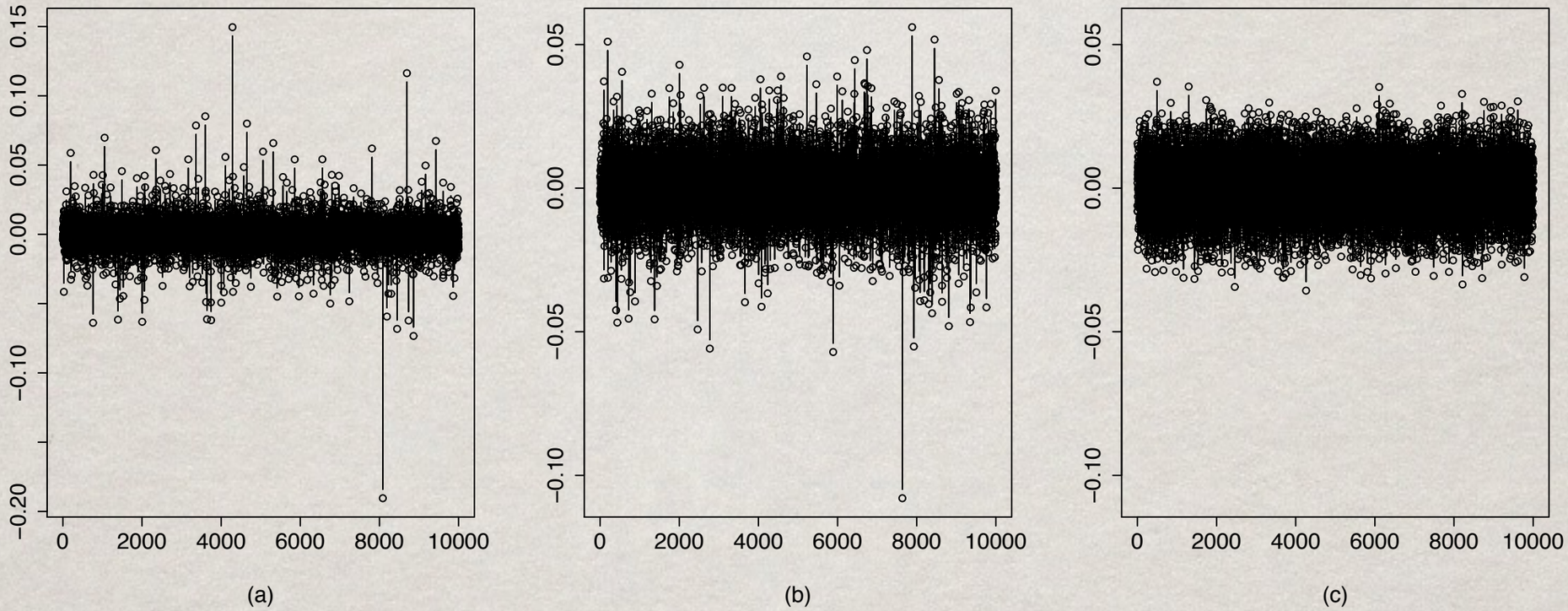


Figure 5.10: *Simulated i.i.d. time series.* (a) Student's t-distribution with degrees of freedom $\nu = 3$. (b) Student's t-distribution with degrees of freedom $\nu = 6$. (c) Gaussian distribution. The mean of the observations is zero and the standard deviation is equal to the standard deviation of the S&P 500 returns-

Autocorrelation

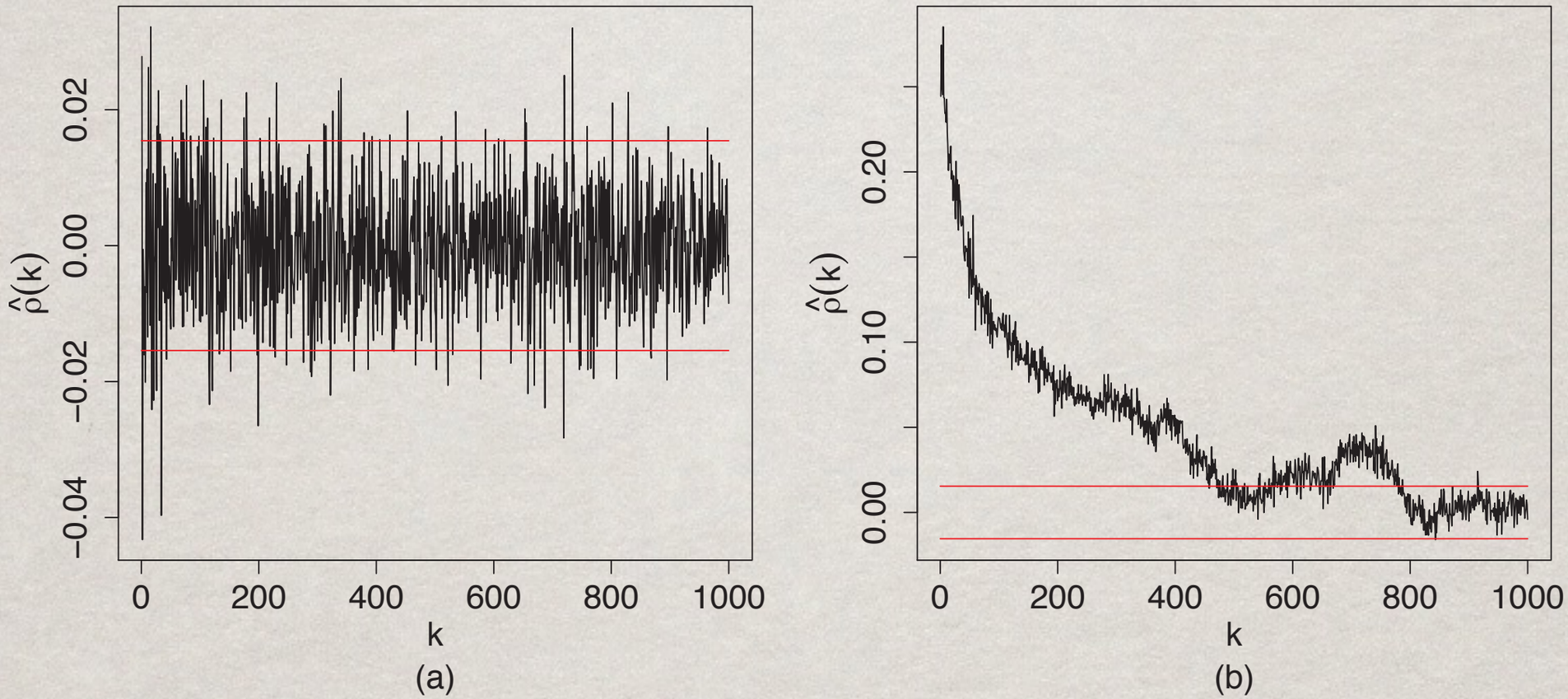


Figure 5.4: *S&P 500 autocorrelation.* (a) The sample autocorrelation function $k \mapsto \hat{\rho}(k)$ of S&P 500 returns for $k = 1, \ldots, 1000$. (b) The sample autocorrelation function for absolute returns.

Heavy tails

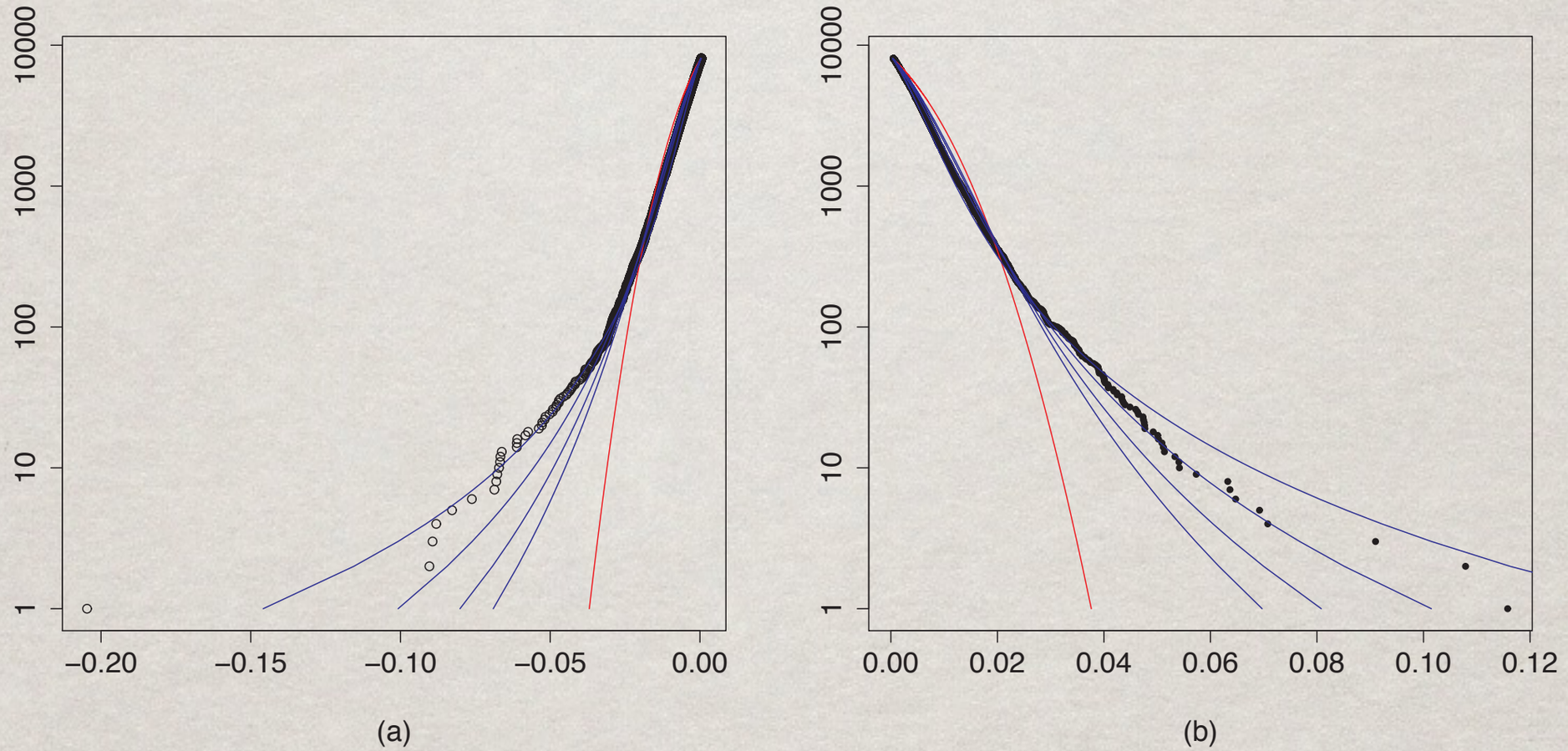


Figure 3.1: *Left and right tail plots.* (a) The left tail plot for S&P 500 returns; (b) the right tail plot. The red curve shows the theoretical Gaussian curve and the blue curves show the Student curves for the degrees of freedom $\nu = 3, 4, 5, 6$.

Heavy tails cont.

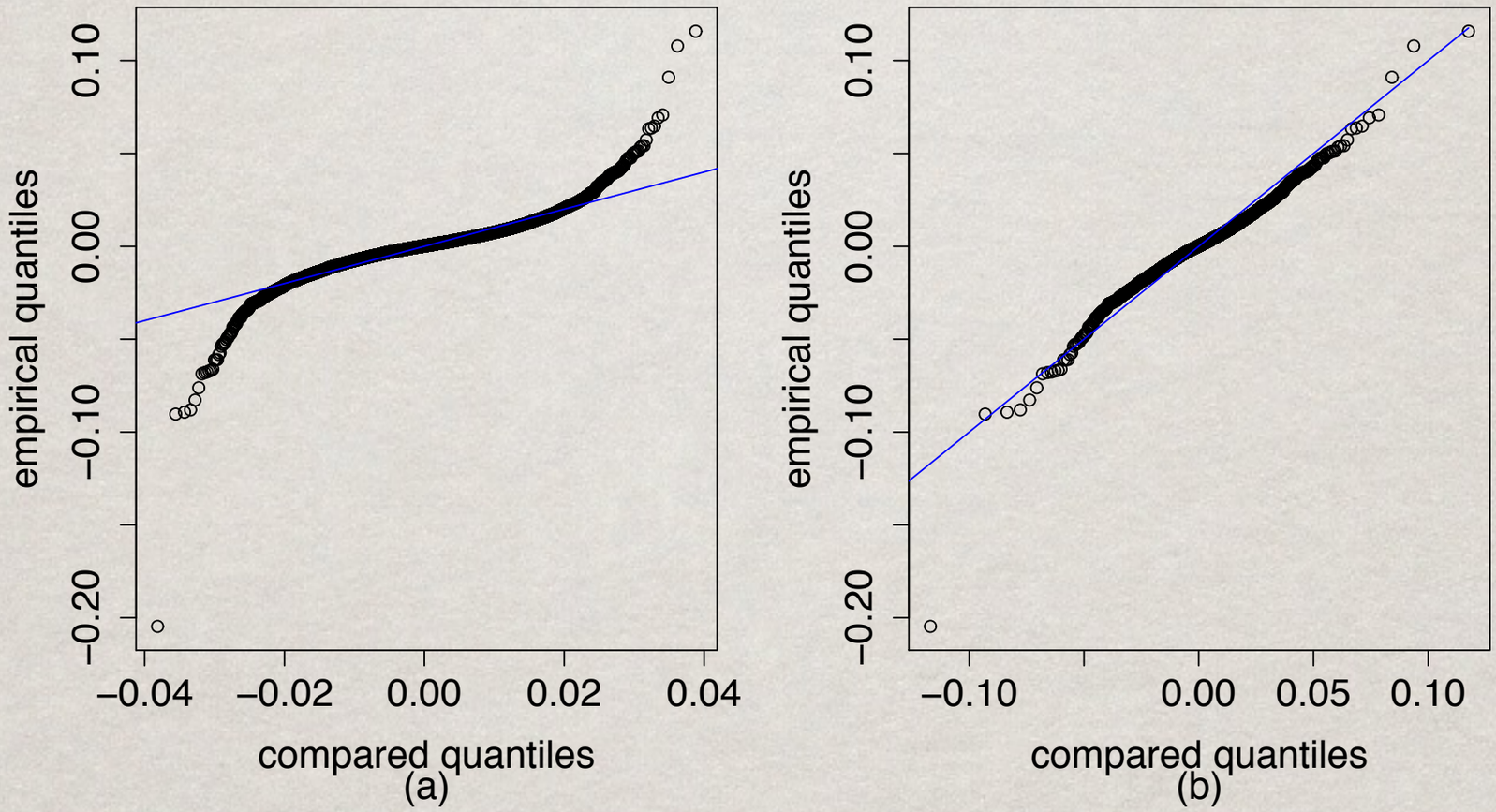
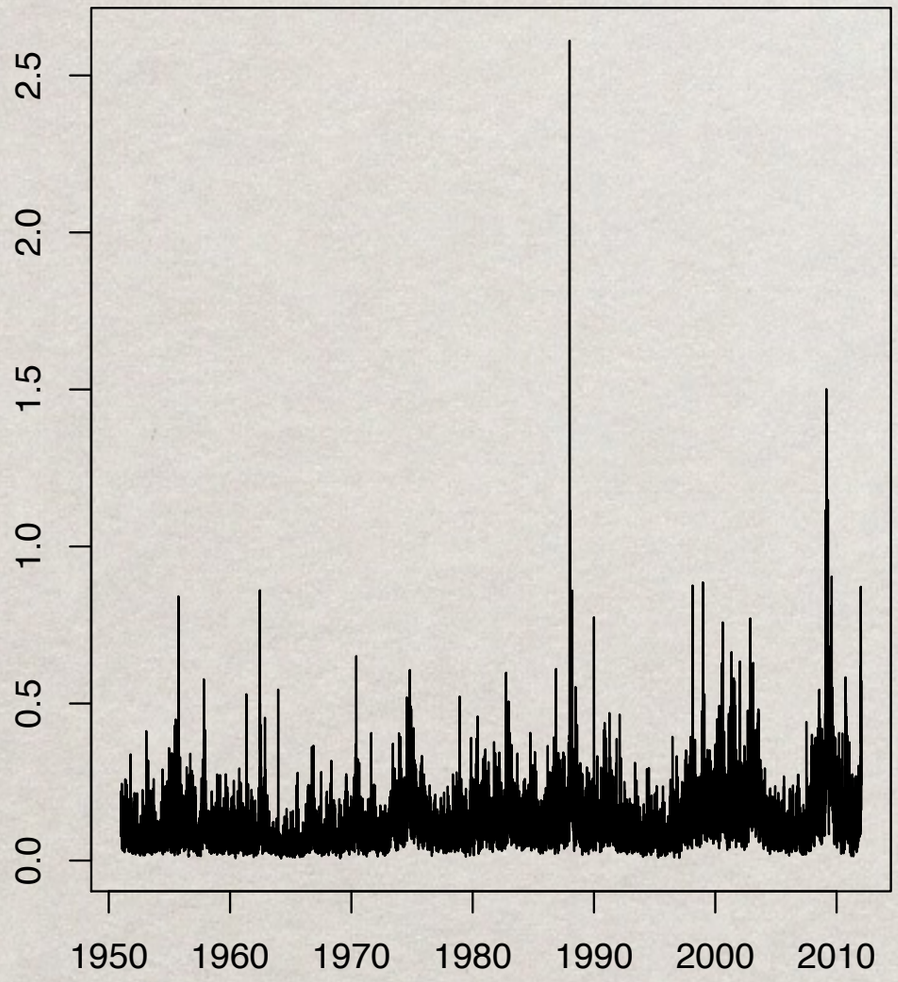
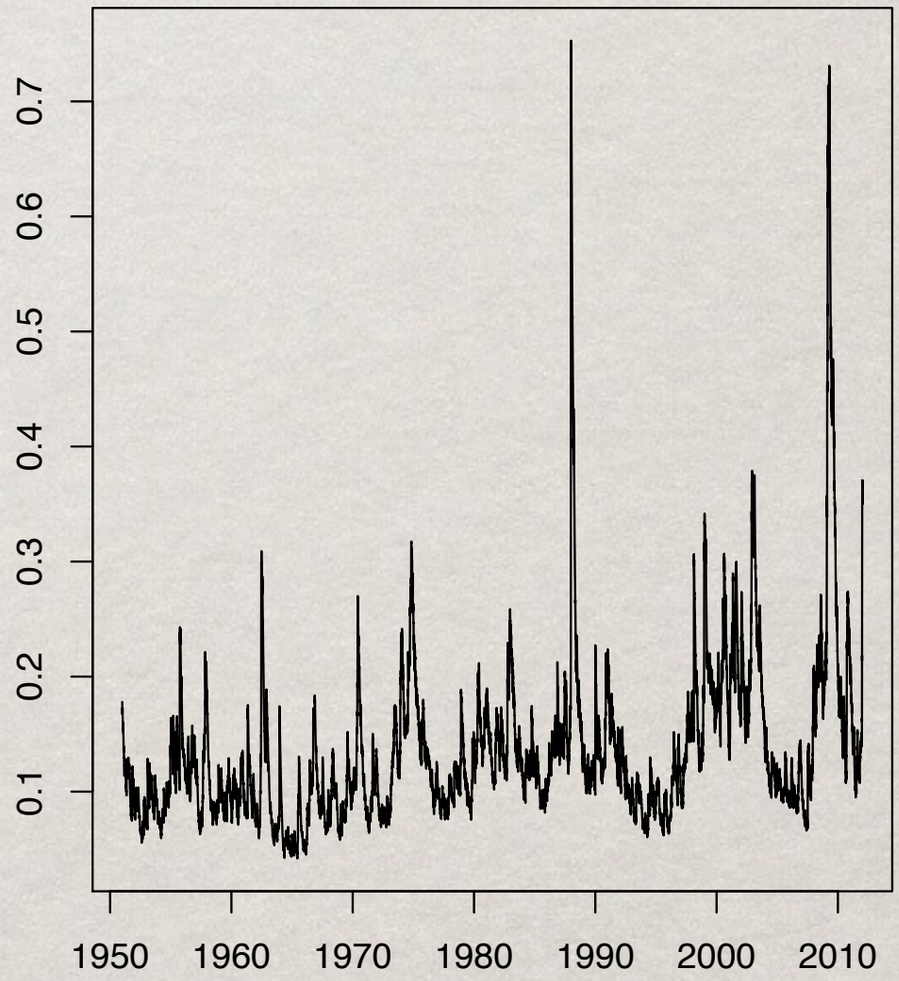


Figure 3.3: QQ-plots of the S&P 500 returns. (a) Comparison to a Gaussian distribution and (b) comparison to a Student distribution with degrees of freedom 5 during the period starting at 1950-01-03 and ending at 2009-08-05.

VOLATILITY: MOVING AVERAGES



(a) h=1



(b) h=25

VOLATILITY: GARCH(1,1) VOLATILITY

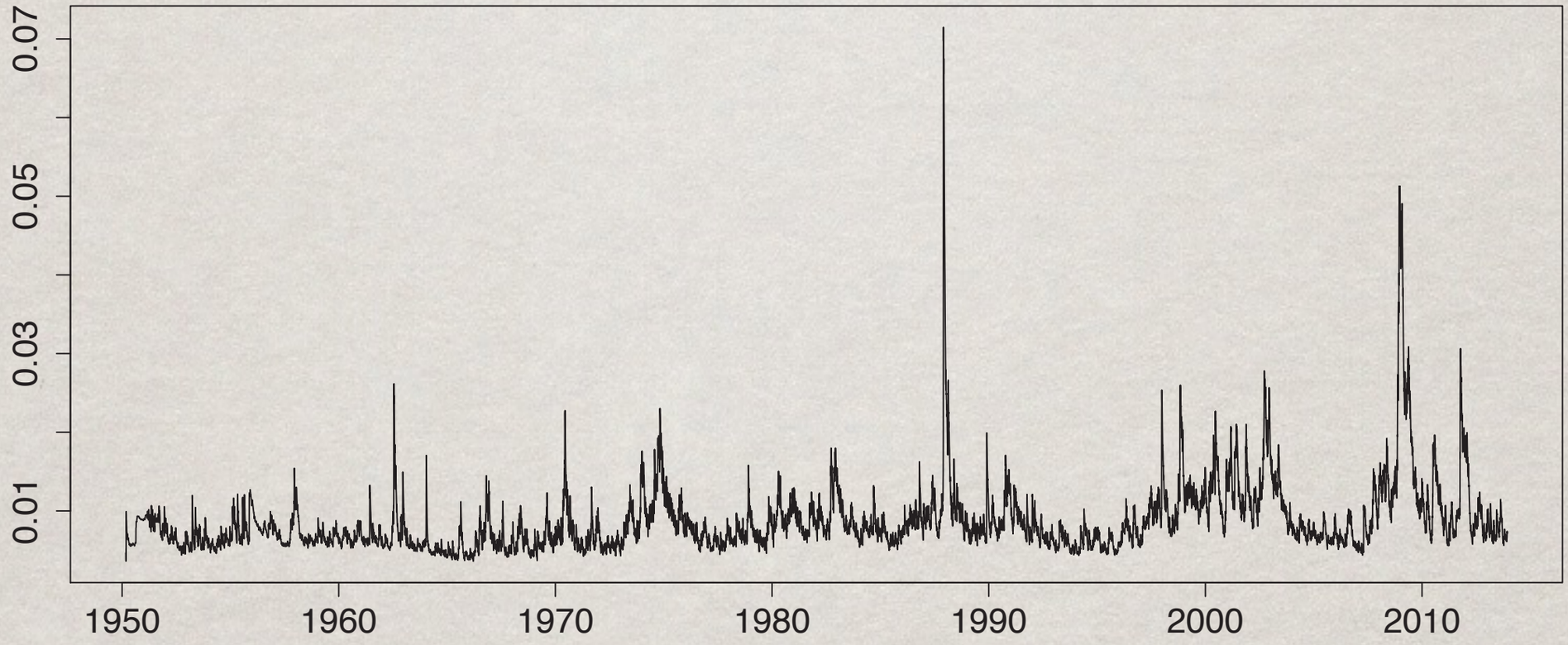
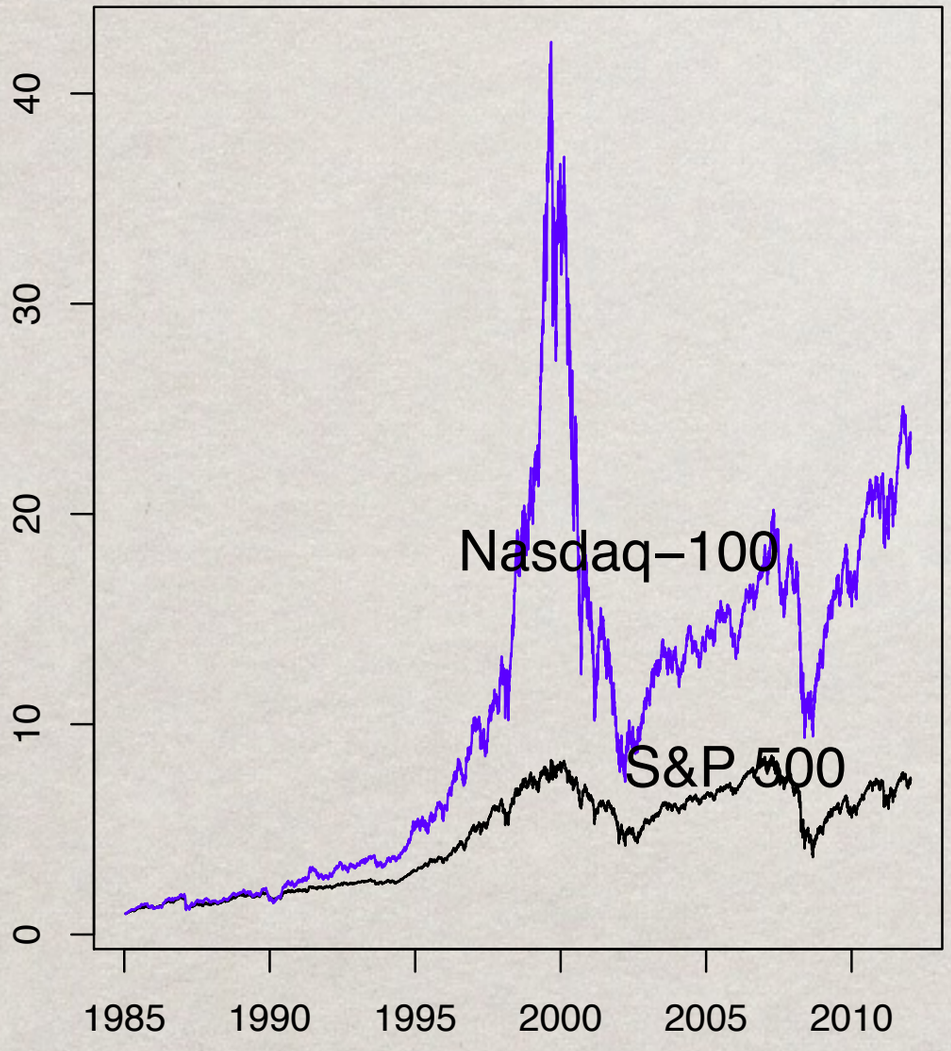
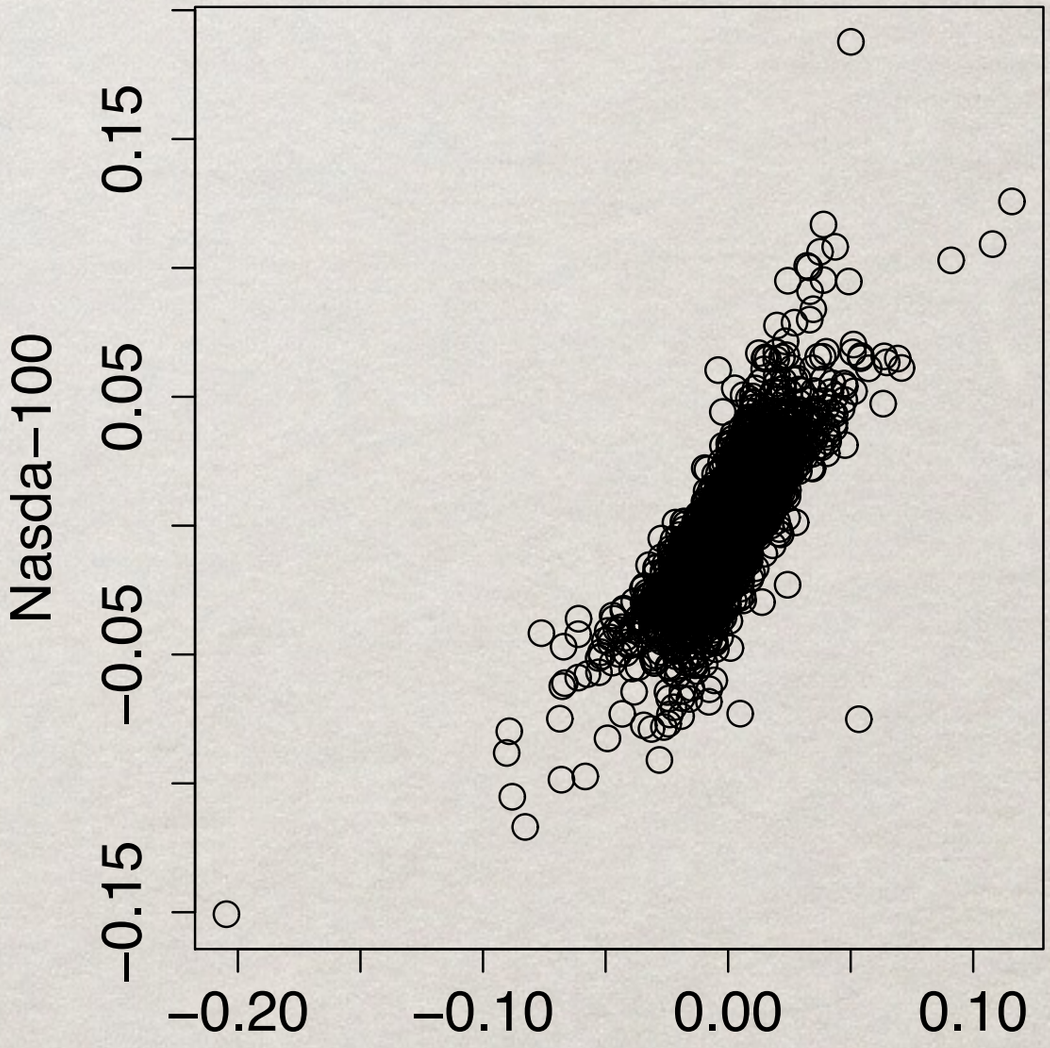


Figure 6.1: *S&P 500 volatility process.* The time series of estimated volatility $\hat{\sigma}_t$ in the GARCH$(1,1)$ model.

DEPENDENCE



(a) Index values



(b) Returns

DEPENDENCE: COPULA

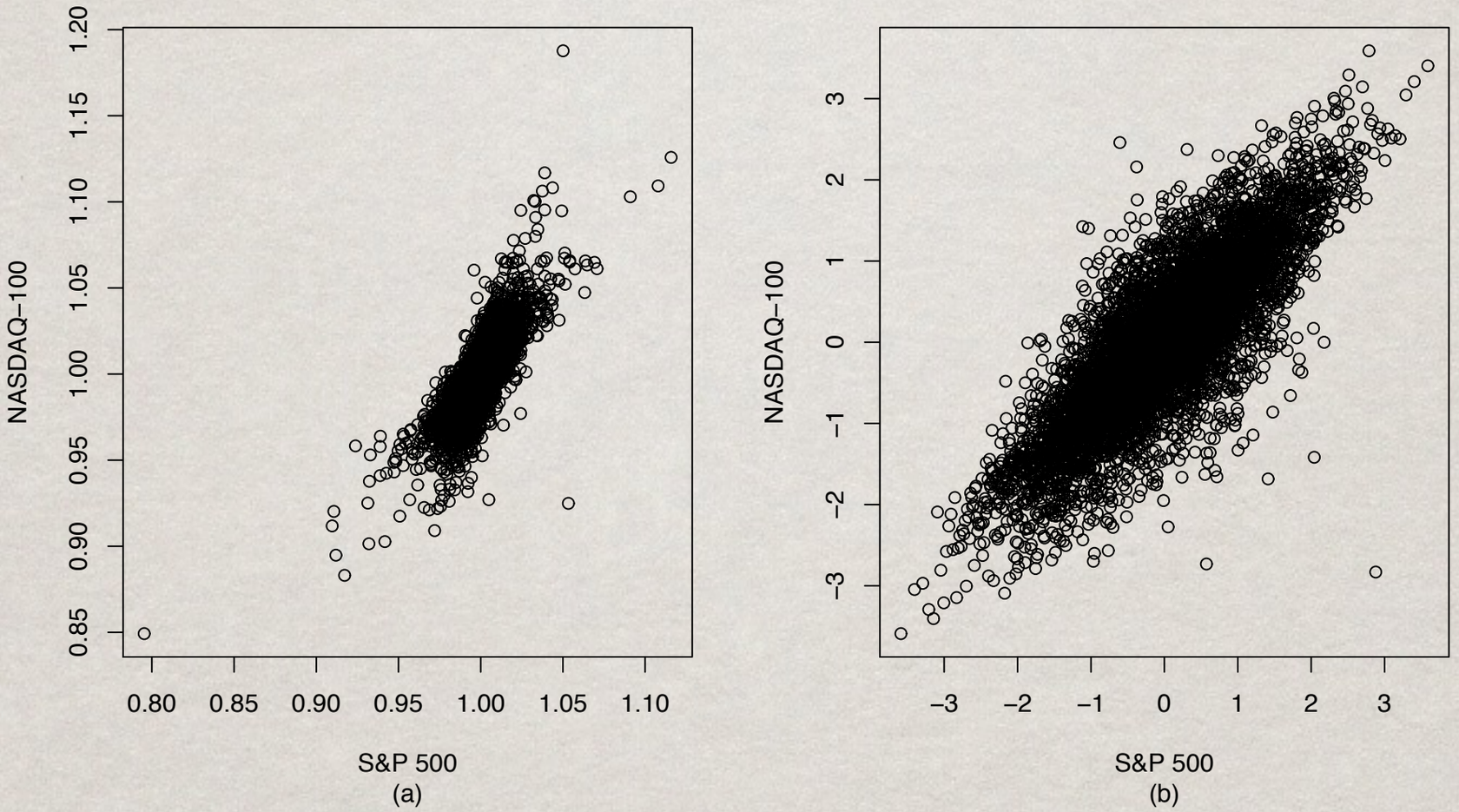
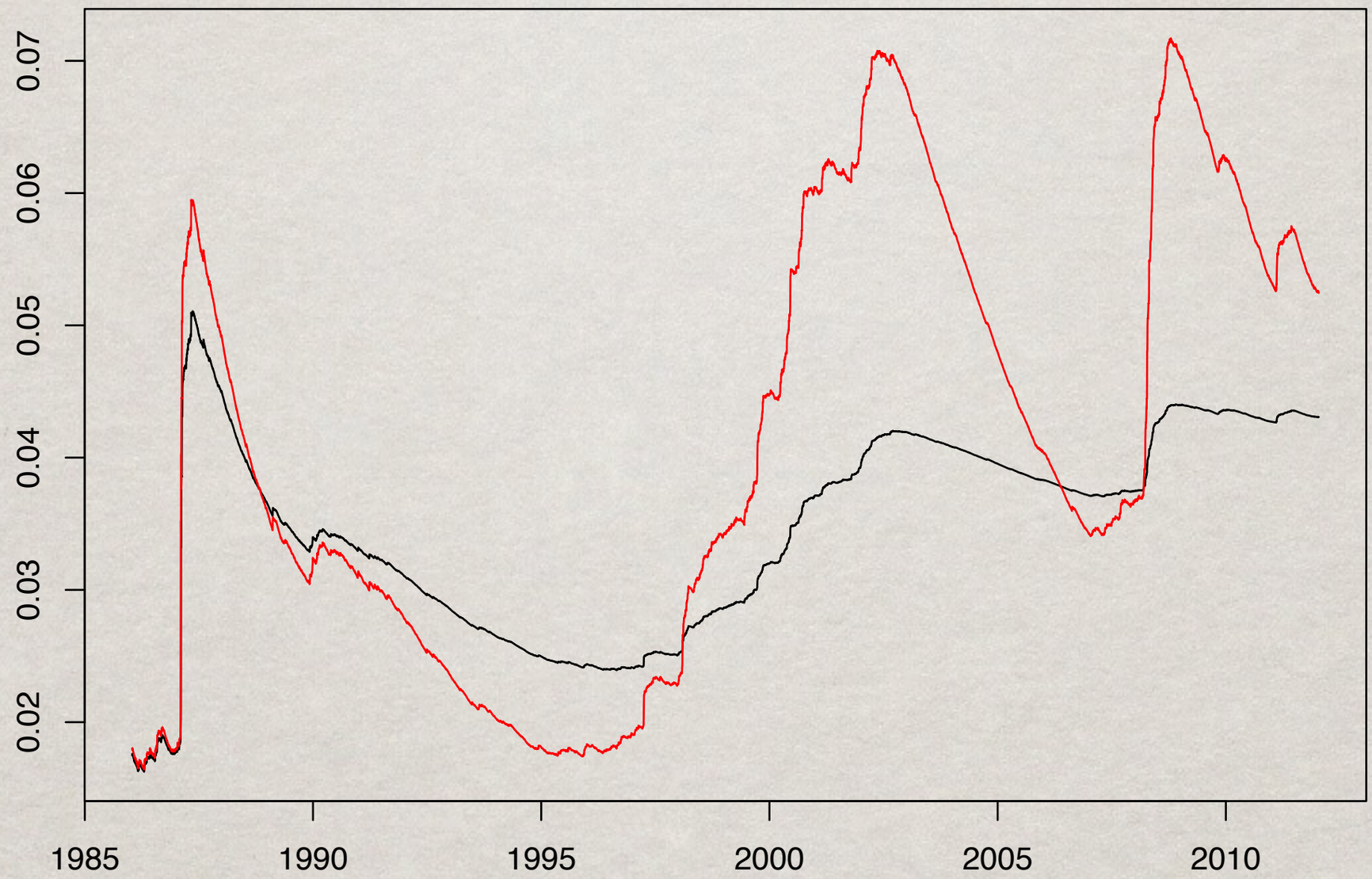


Figure 4.1: Scatter plots of the returns of stock indeces S&P 500 and NASDAQ-100 during the period from 1 October 1985 to 17 August 2009.

CORRELATION PROCESS