VISUALIZATION

Review of Visualizations

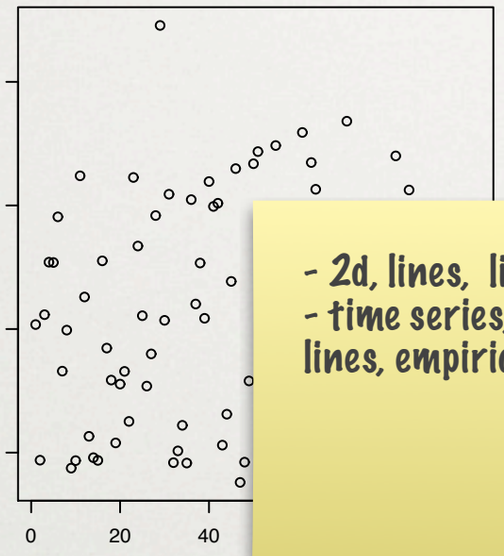
VISUALIZATION OF DATA

Visualization of 1D data

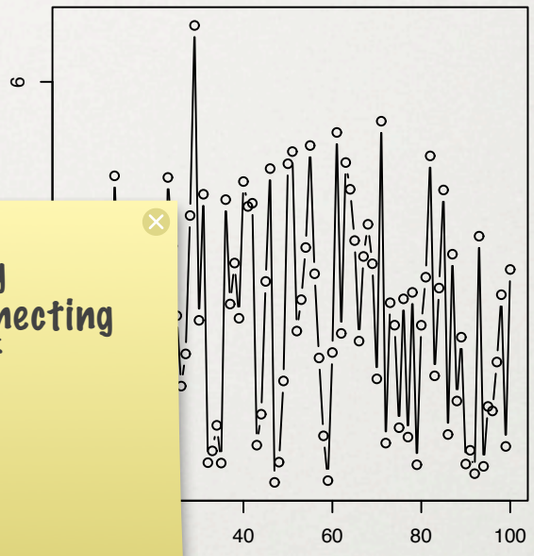
1D data

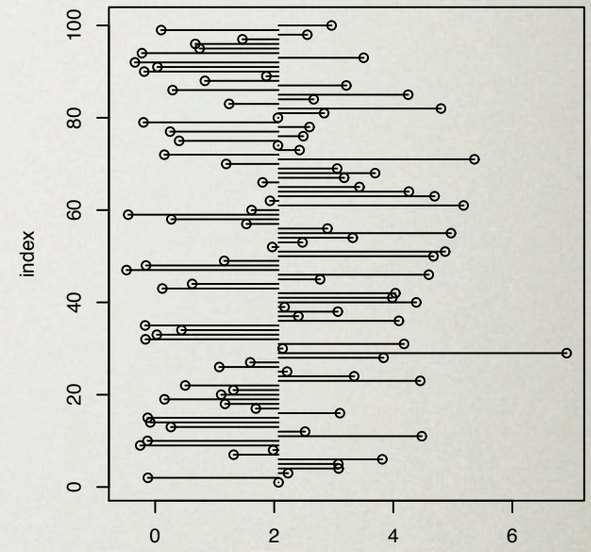
- We want to visualize a vector of real numbers
- 1.2, 2.0, -10.0, ...

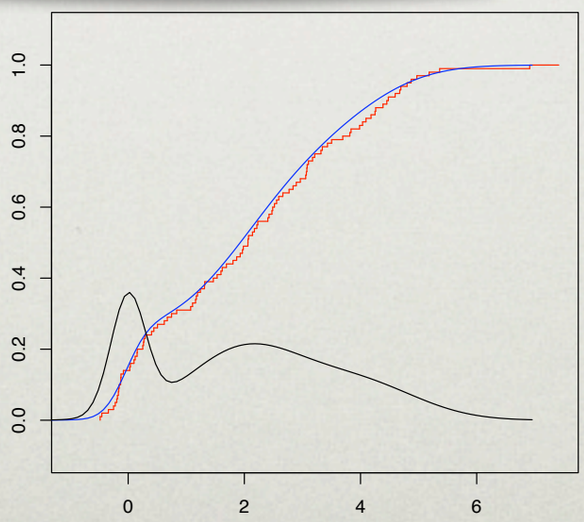
1D visualizations



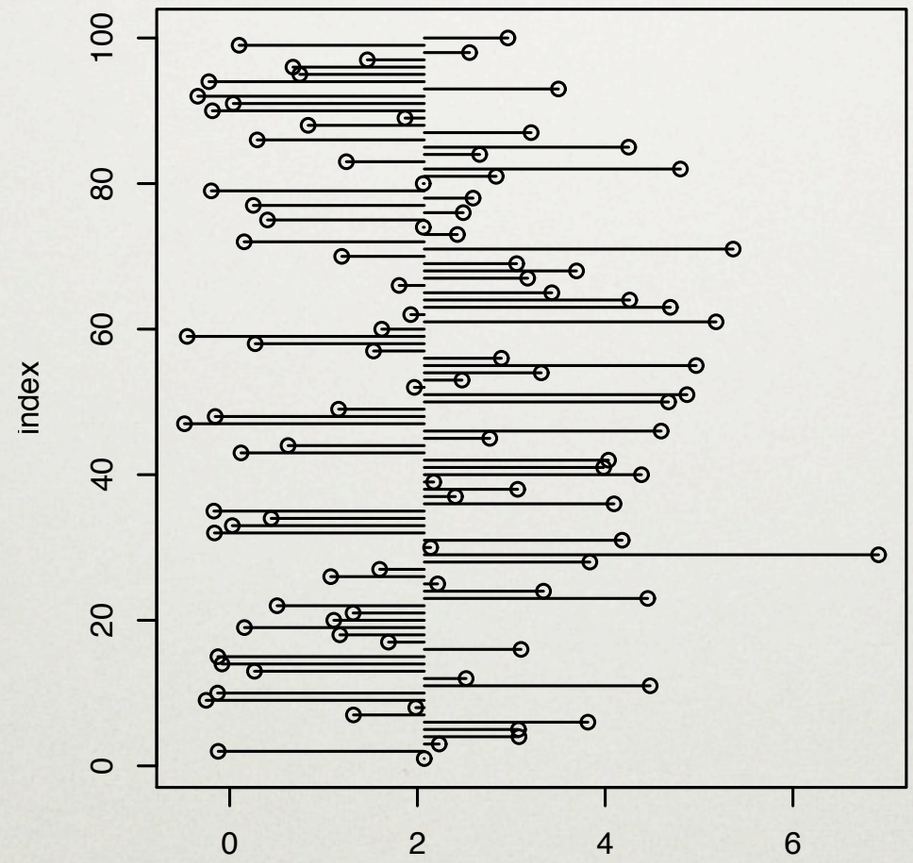
- 2d, lines, lifting
- time series, connecting lines, empirical df



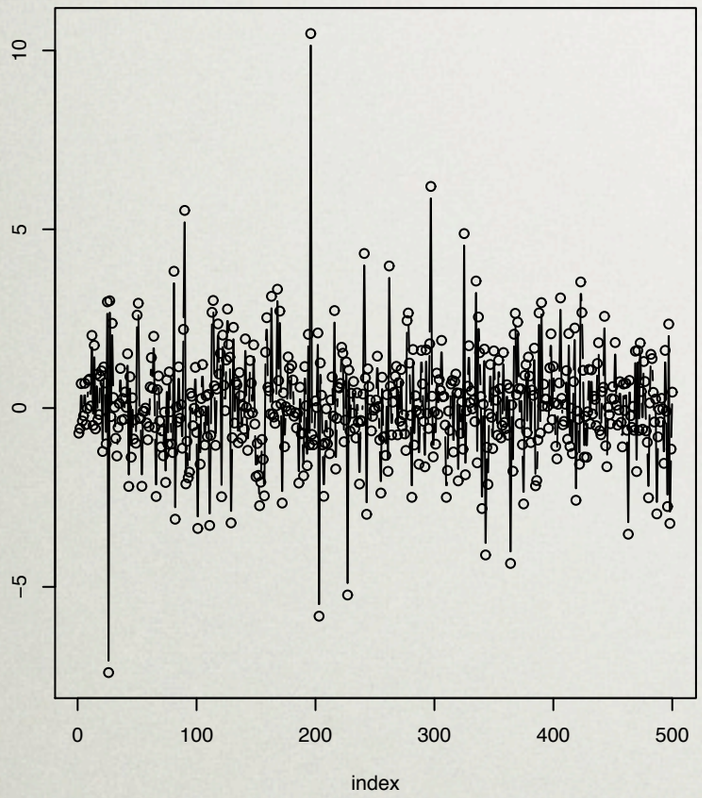


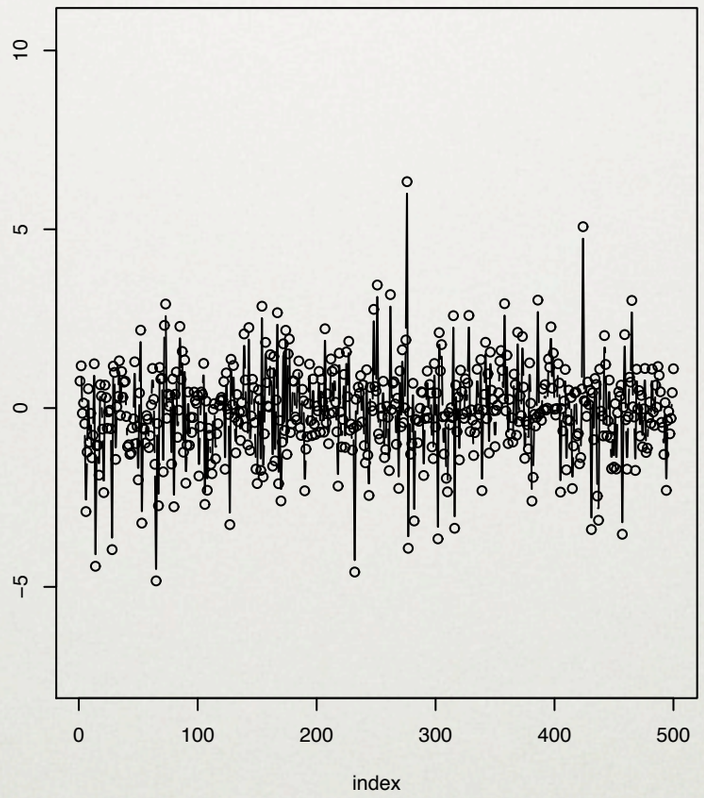


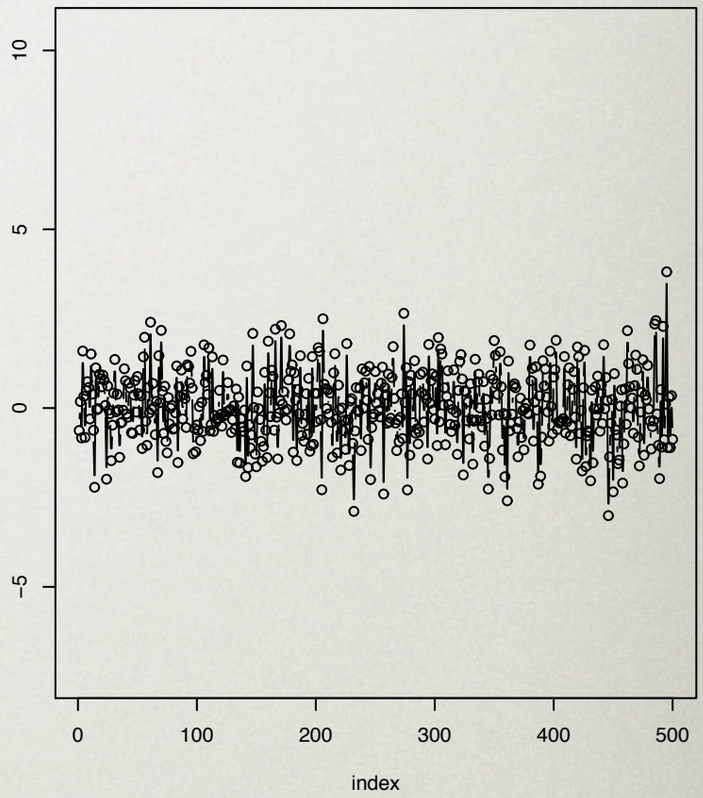
Level Plot



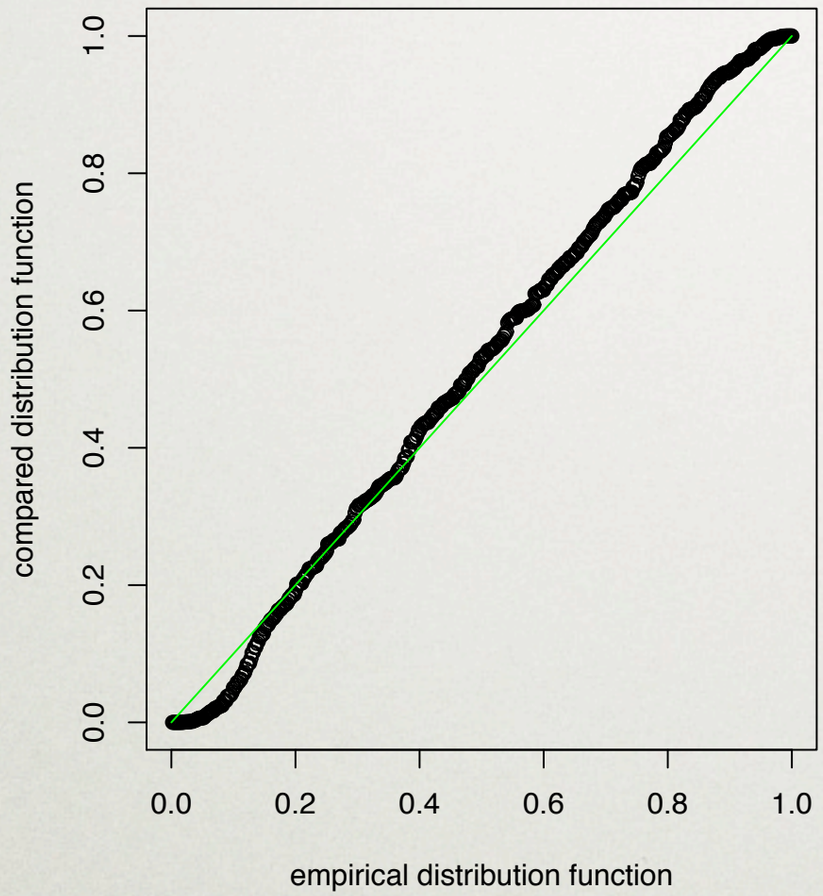
Time series plot

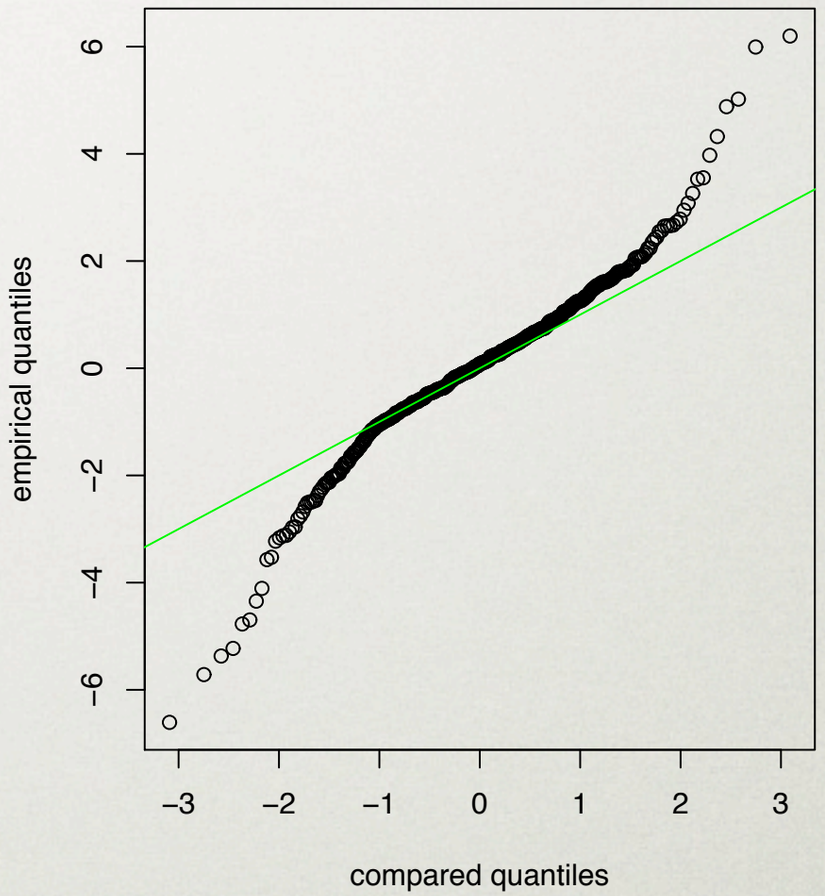




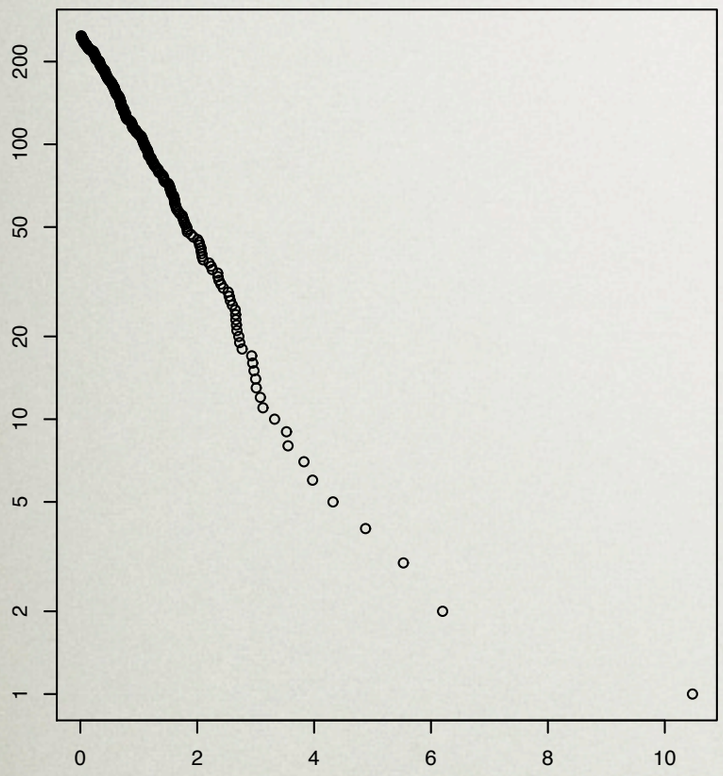


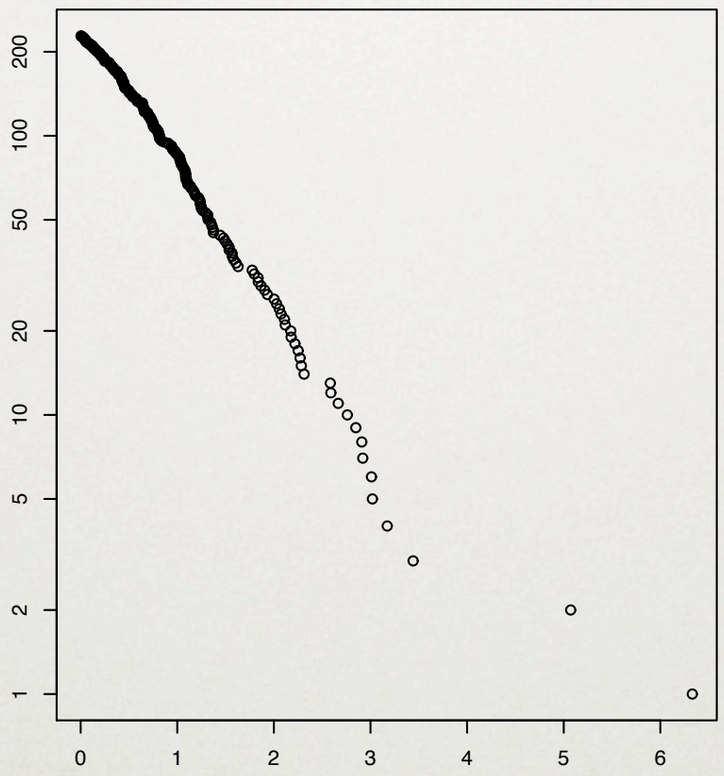
PP-plot and QQ-plot

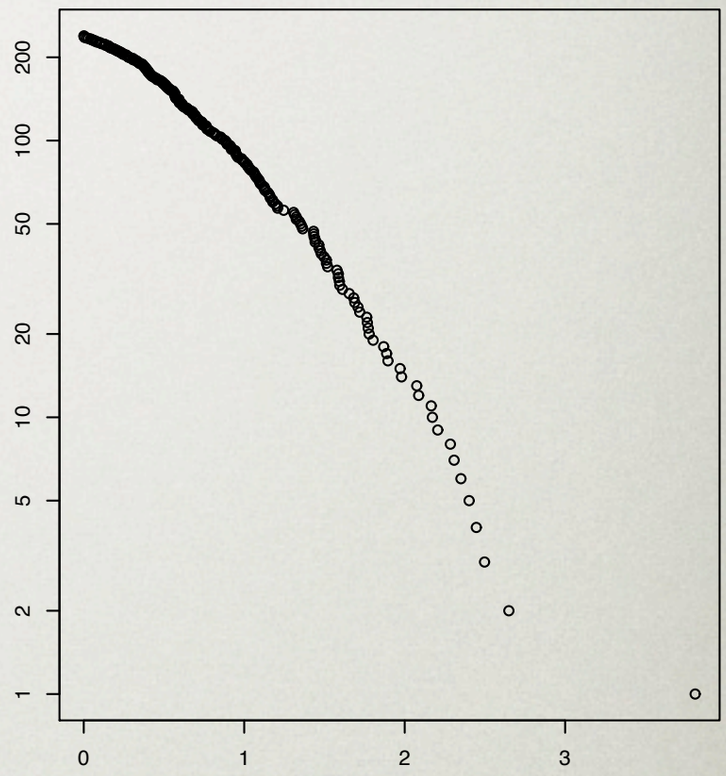




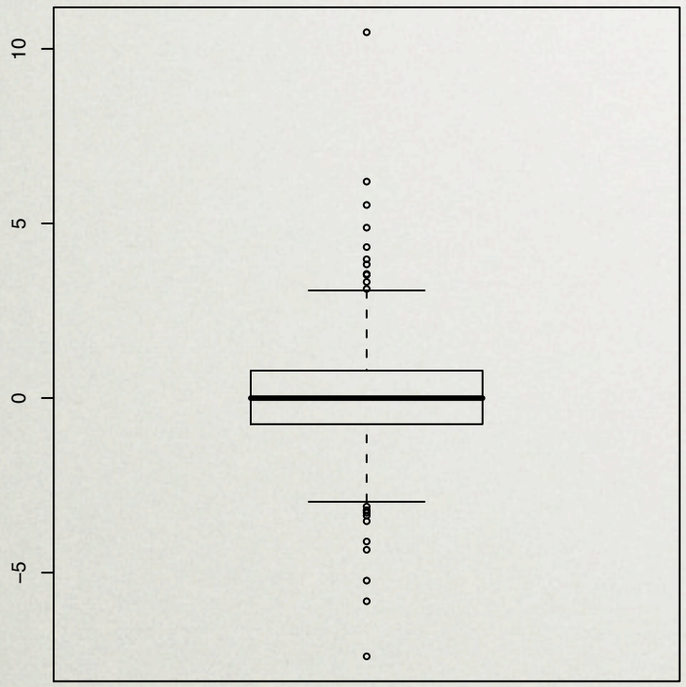
Right tail plot

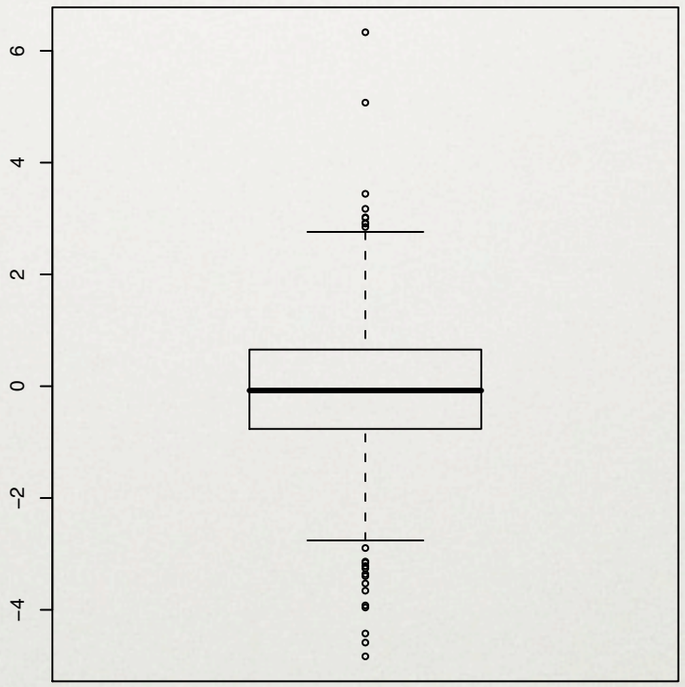


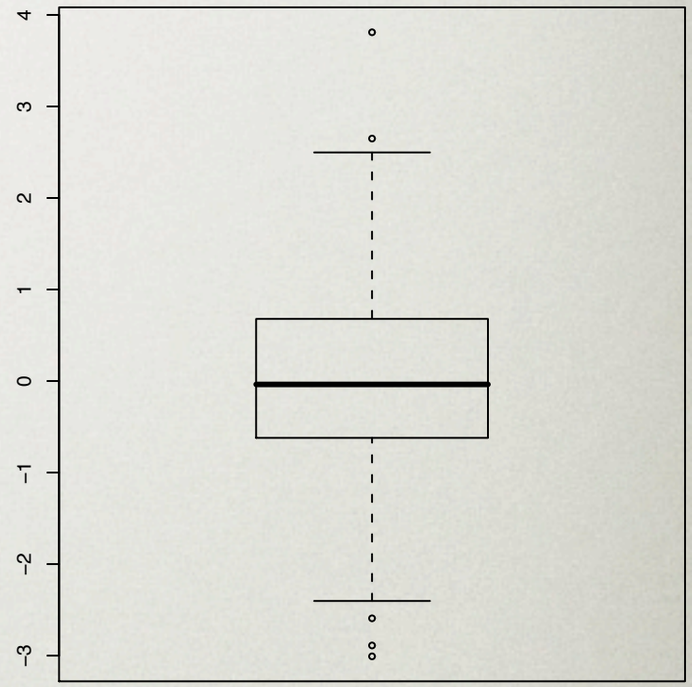




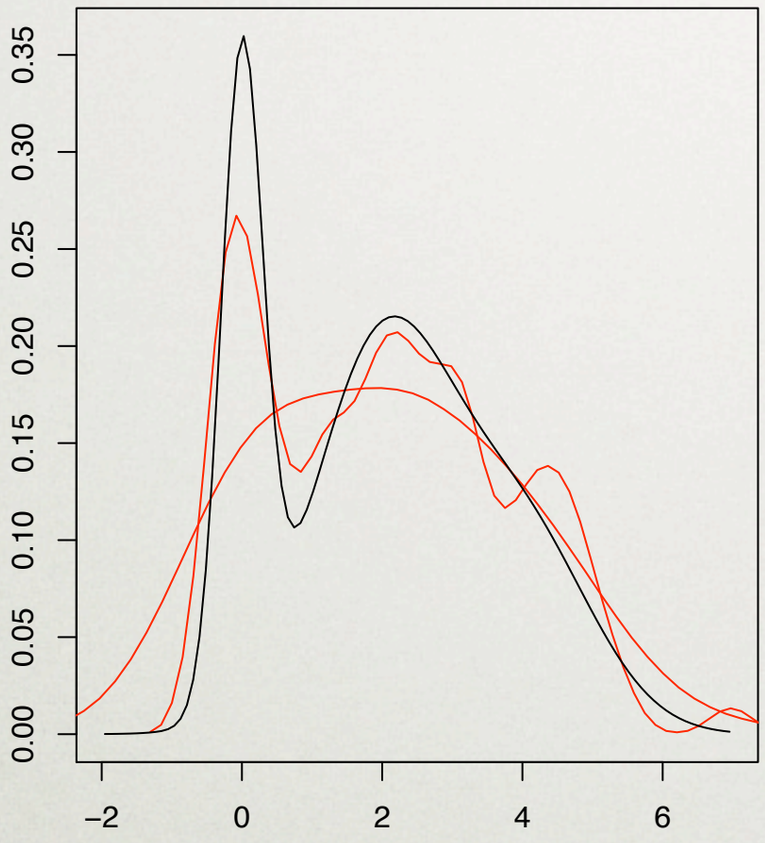
Box plots (Tukey)



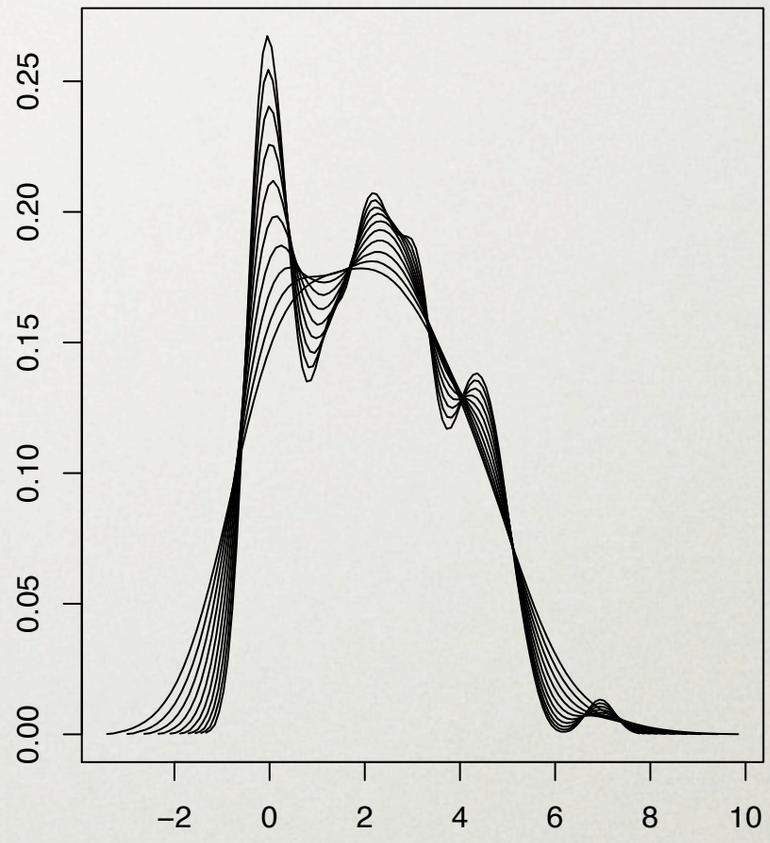




Kernel estimates

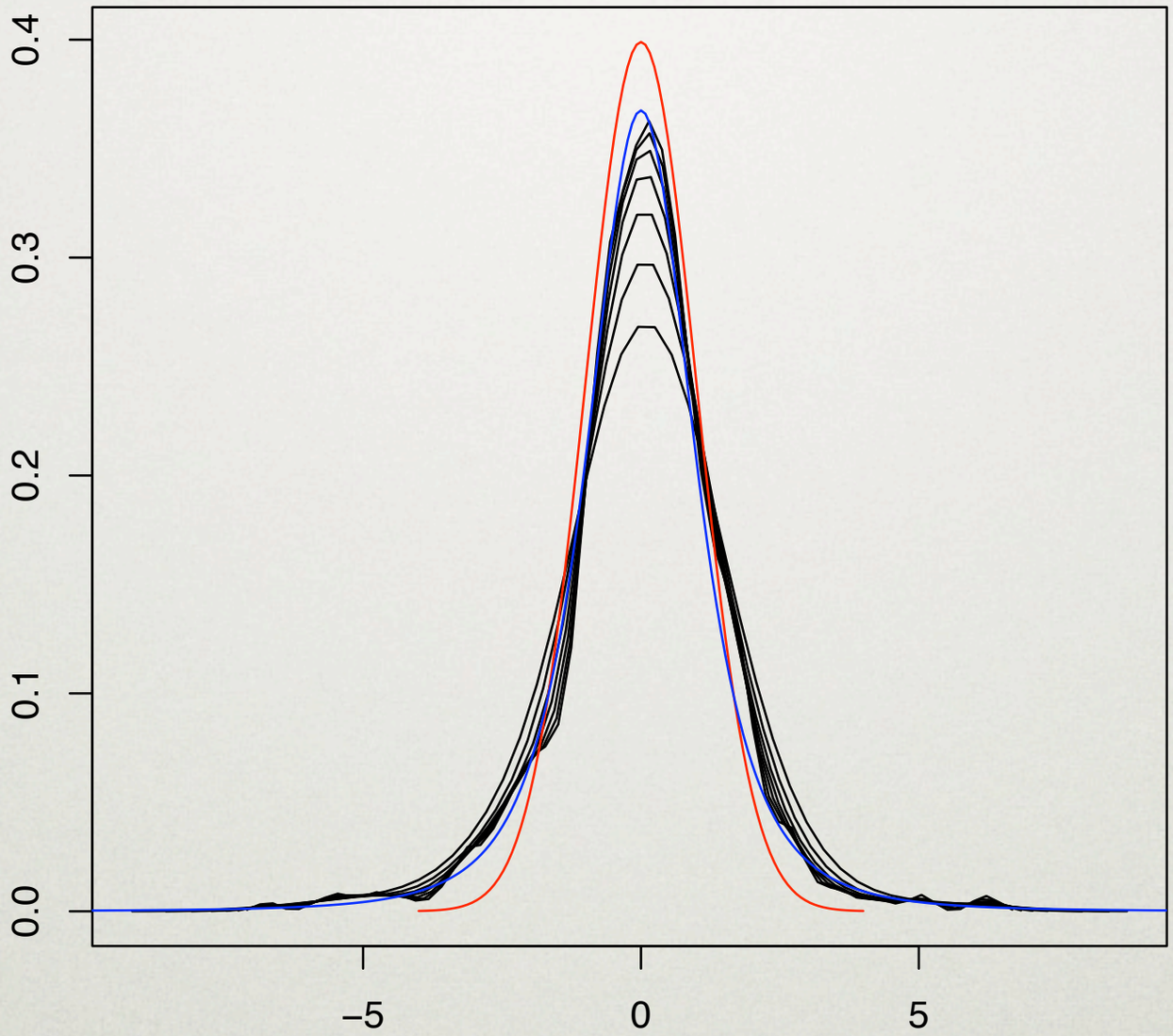


2 kernel estimates

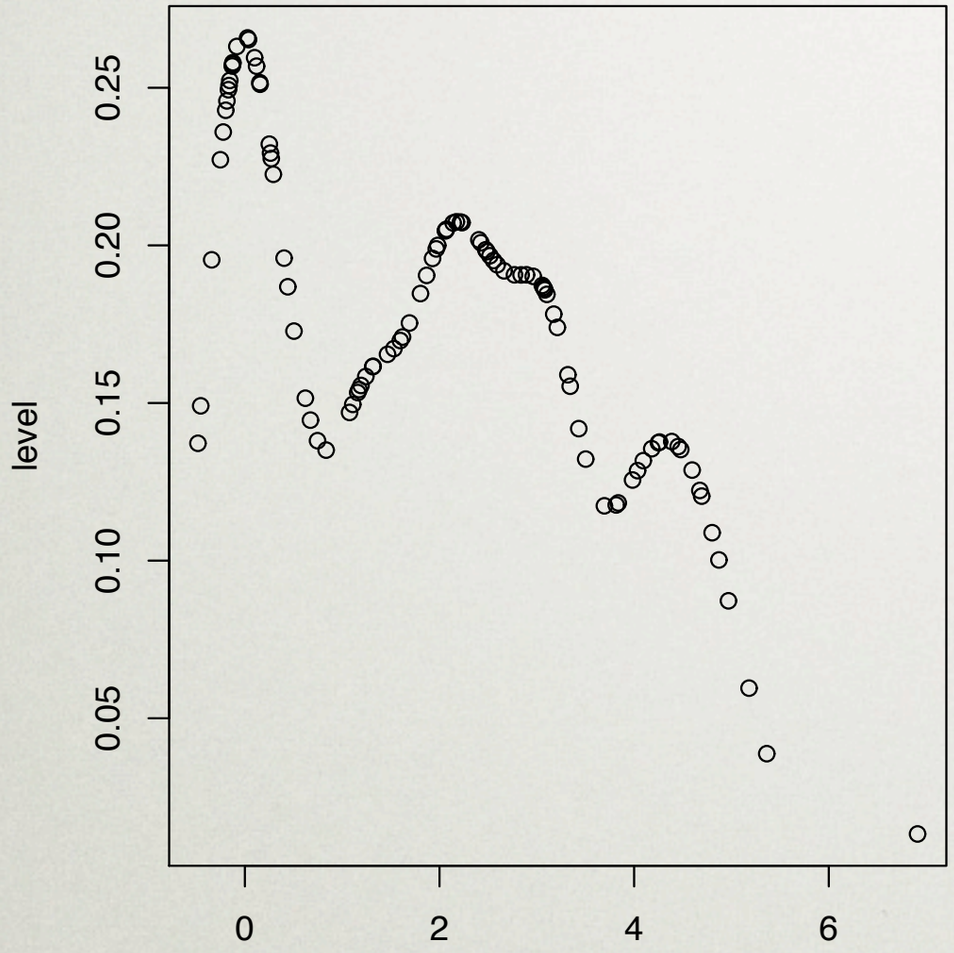


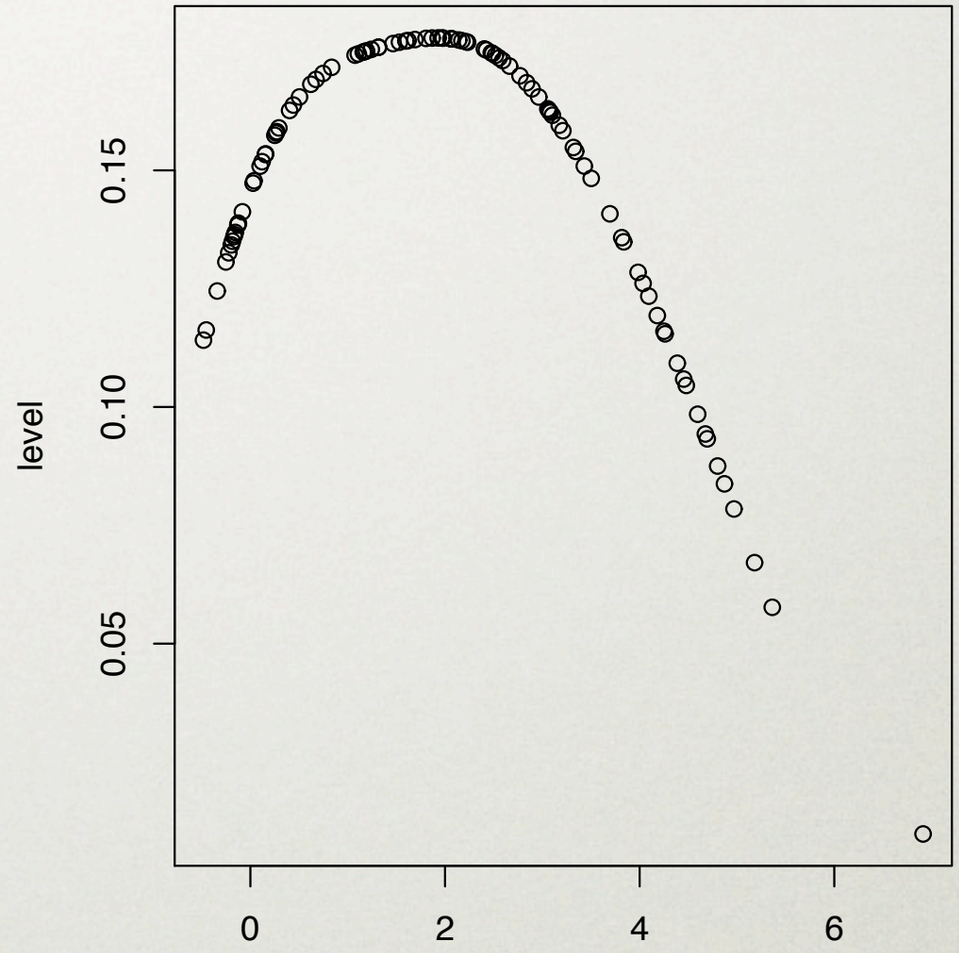
a scale of kernel estimates

Scale of kernel estimates



1D scatter plots

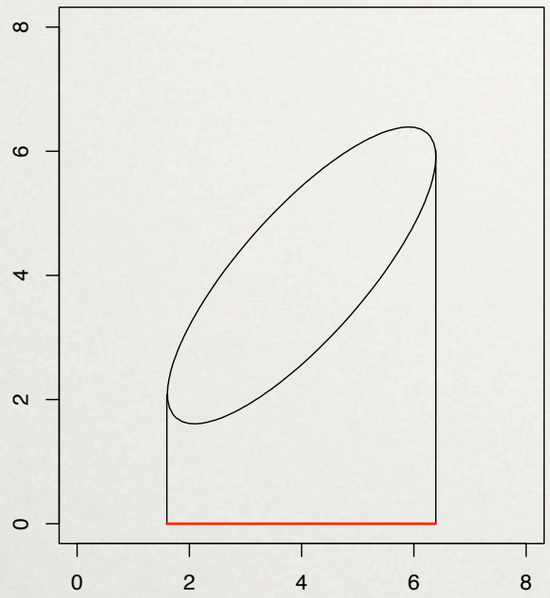


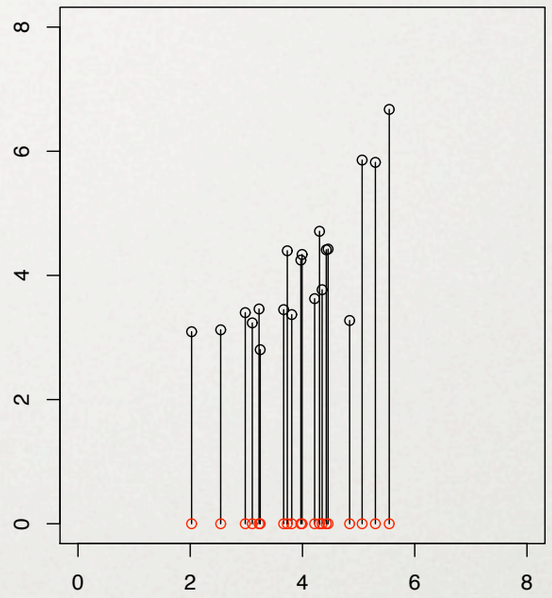


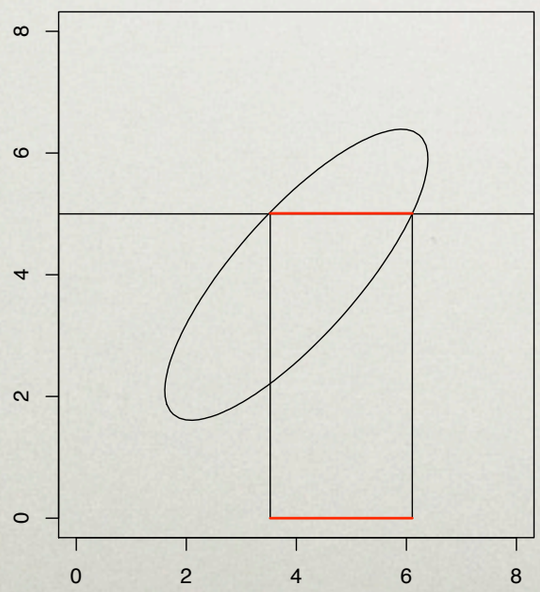
Visualization of Multivariate data

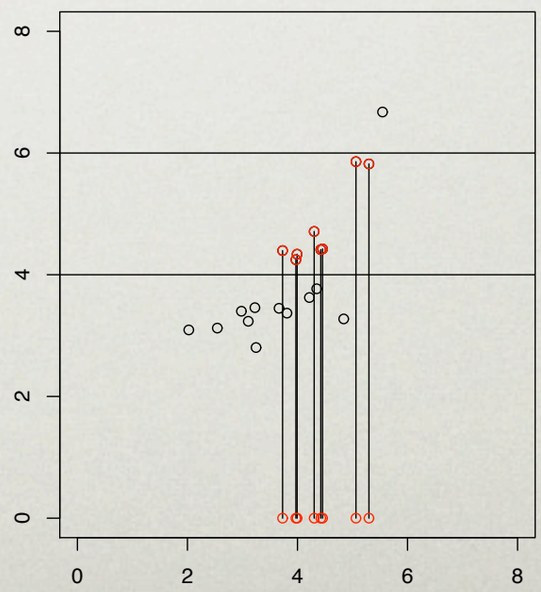
Projections, slices, and scatter plots

Projection and Slice

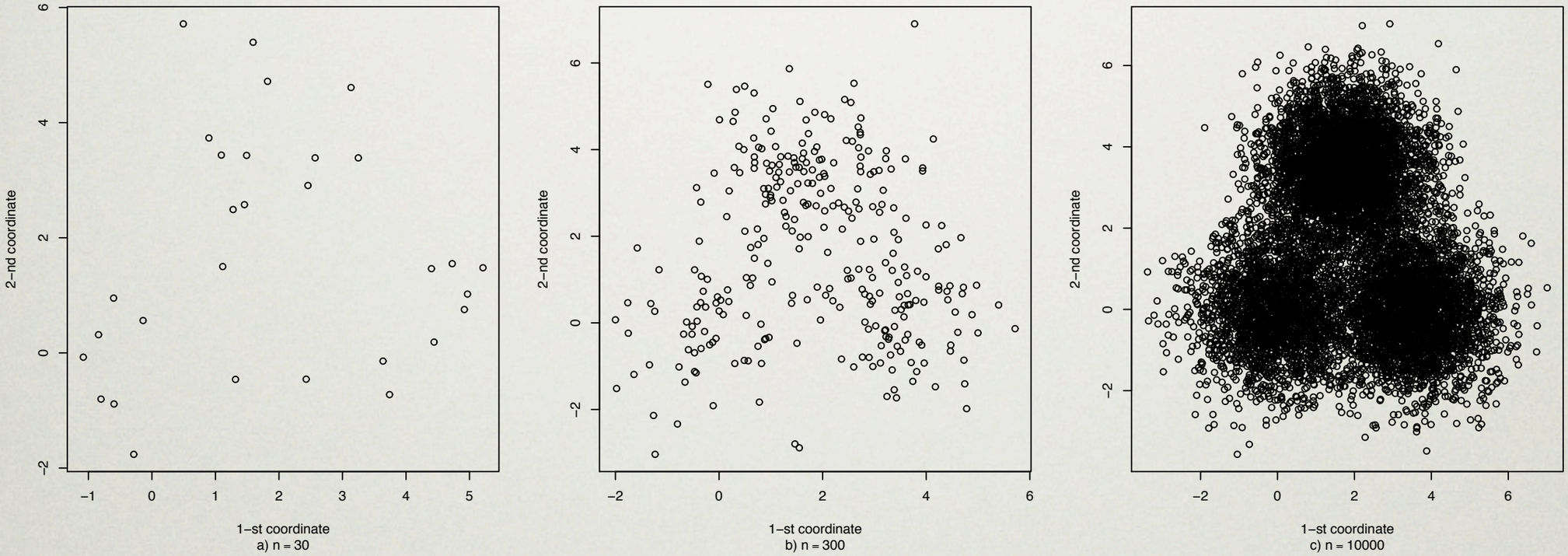






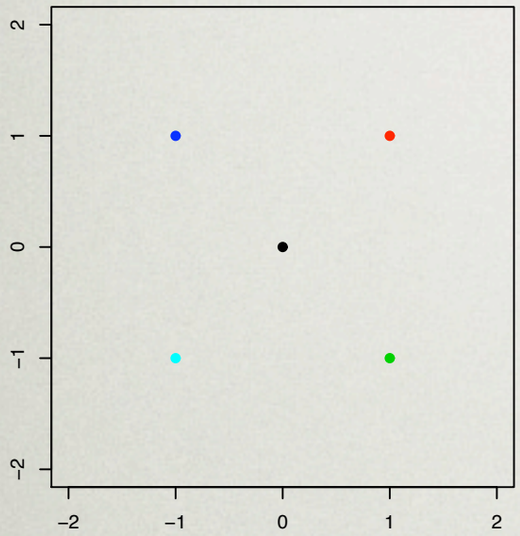


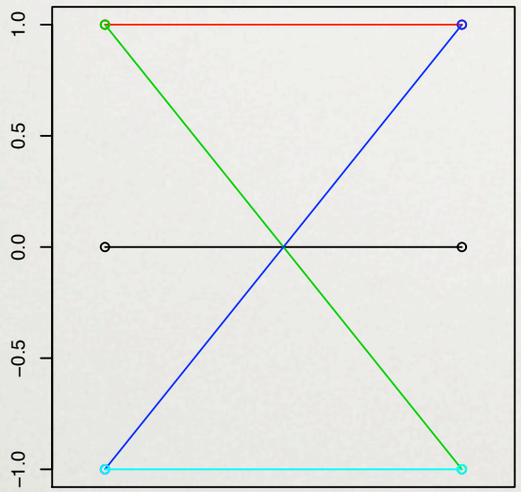
Scatter plots different sample sizes

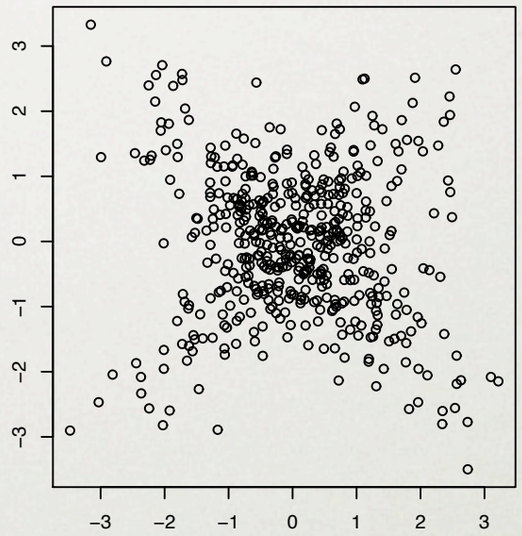


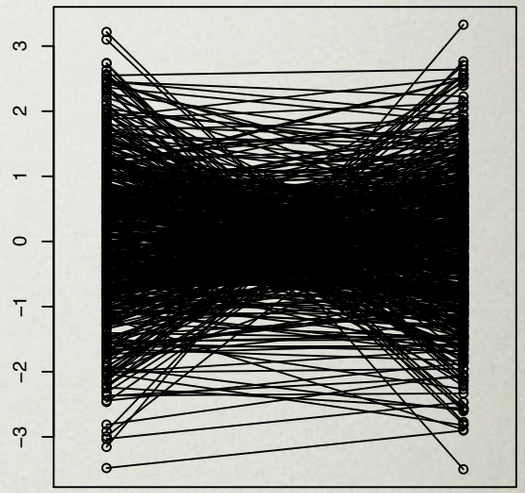
Parallel Coordinate Plots

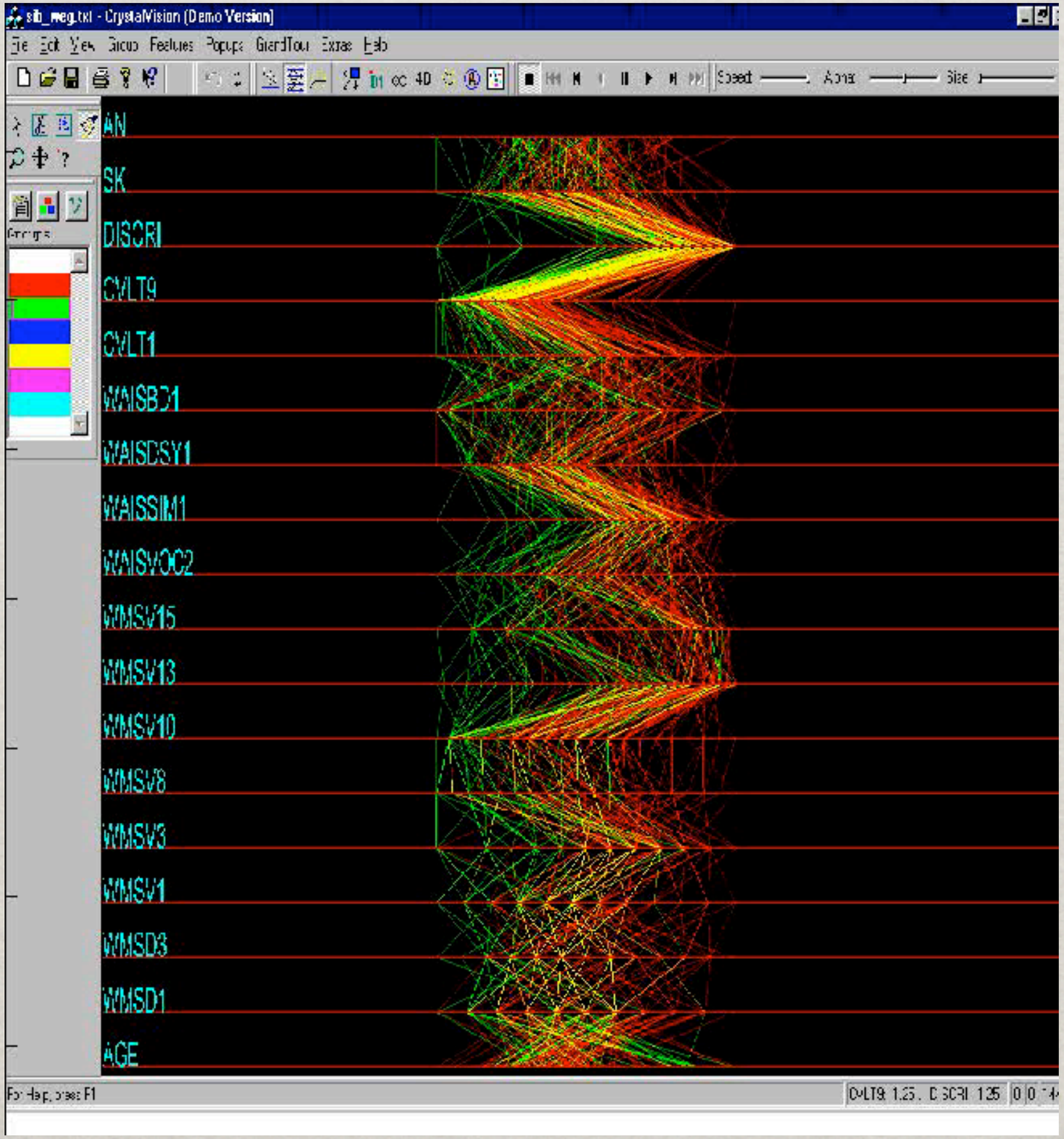
Parallel coordinate plots



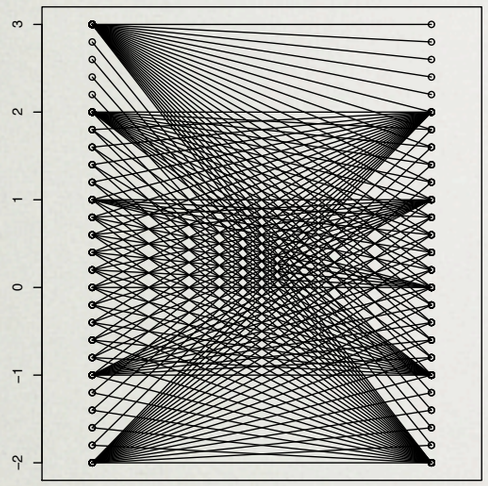


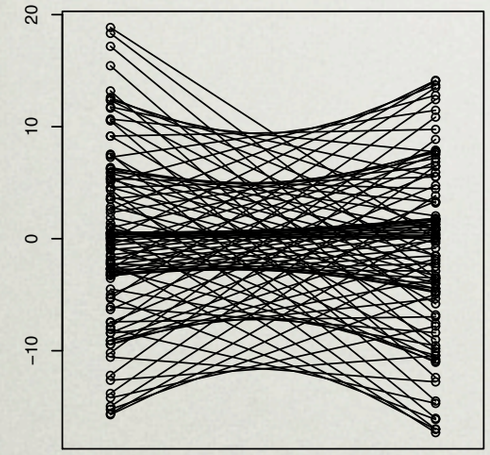


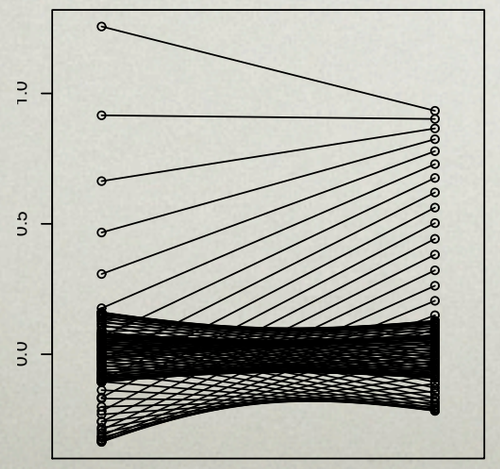


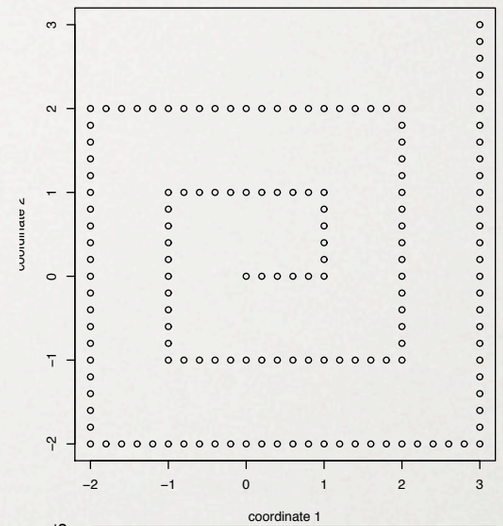

slb_reg.txt - CrystalVision (Demo Version)
File Edit View Group Features Popups GlandTour Extras Help
Speed
Accel
Size
AN
SK
DISCRI
CVLT9
CVLT1
WAISBD1
WAISDSY1
WAISSIM1
WAISVOC2
WMSV15
WMSV13
WMSV10
WMSV8
WMSV3
WMSV1
WMSD3
WMSD1
AGE
For Help, press F1

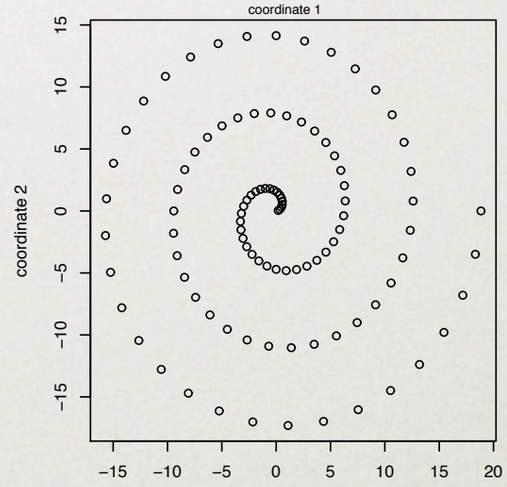
Examples

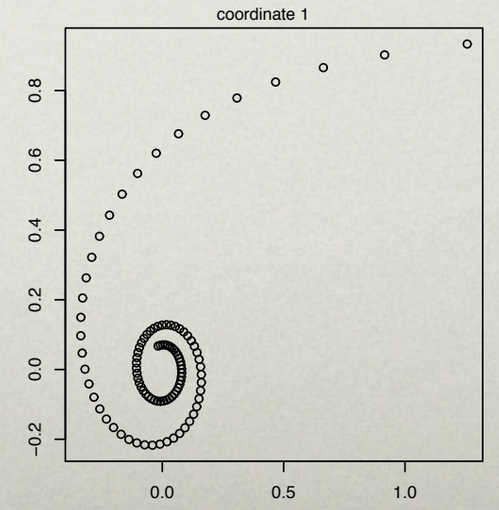

3
2
1
0
−1
−2


20
10
0
−10


1.0
0.5
0.0


3
2
1
0
−1
−2
−2
−1
0
1
2
3
coordinate 1


coordinate 1
coordinate 2
15
10
5
0
−5
−10
−15
−15
−10
−5
0
5
10
15
20

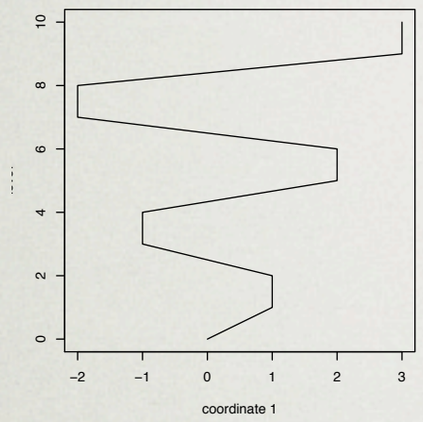

coordinate 1
0.8
0.6
0.4
0.2
0.0
−0.2
0.0
0.5
1.0

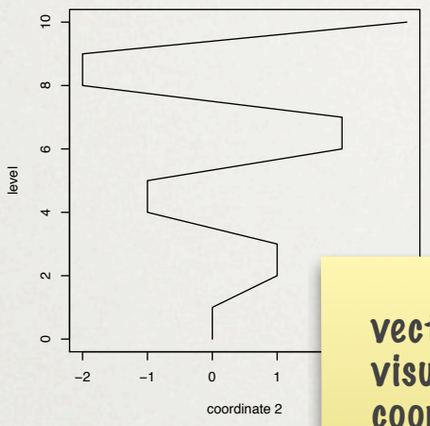
Parallel level plots

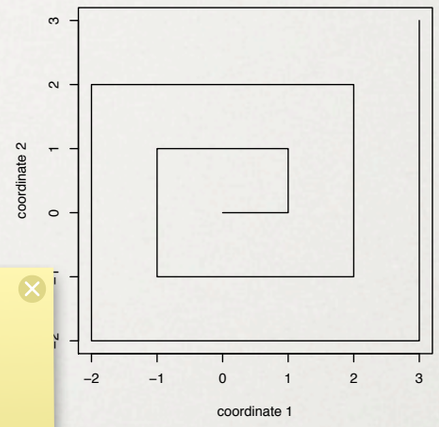
Definition

Definition 2.2 Parallel level plot *of data* $x^1, \ldots, x^n \in \mathbf{R}^d$, *associated with a level mapping* $\mathrm{level}(x^i) \in \mathbf{R}$, *consist of* d *windows, where the* k*-th window,* $k = 1, \ldots, d$, *shows a scatter plot of points* $(x_k^i, \mathrm{level}(x^i))$, $i = 1, \ldots, n$.

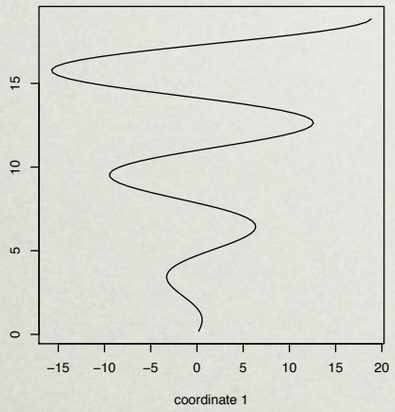
Curves

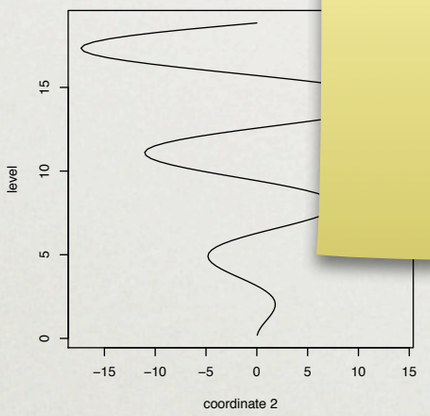


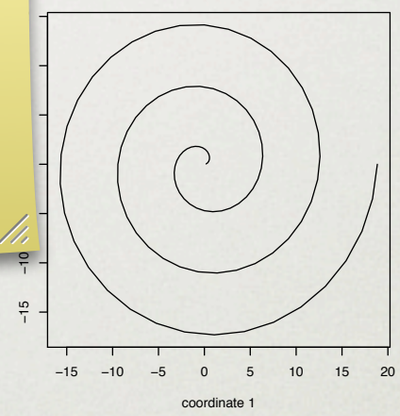


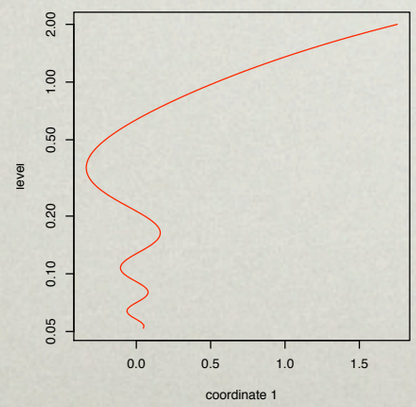


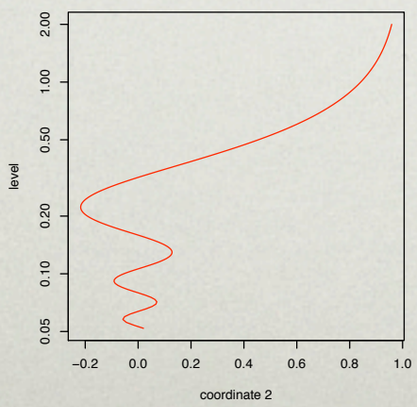
vector valued function is visualized through its coordinate functions

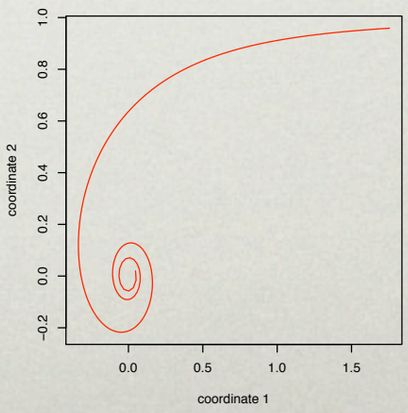




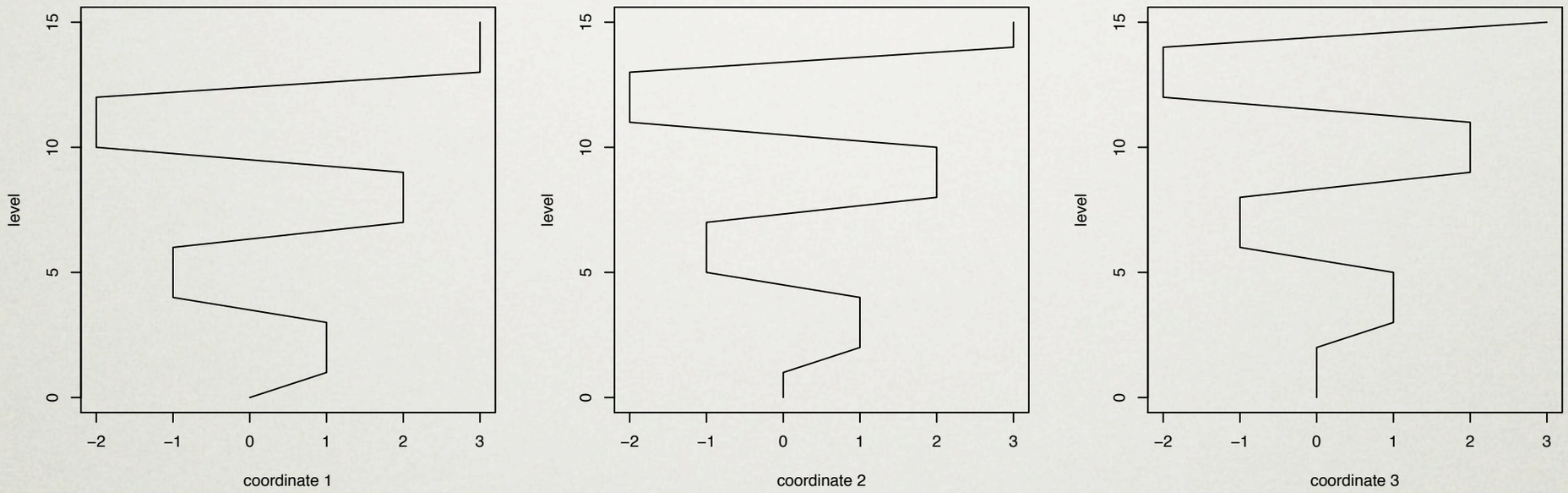




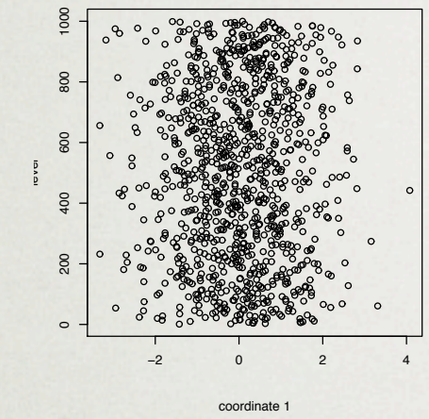


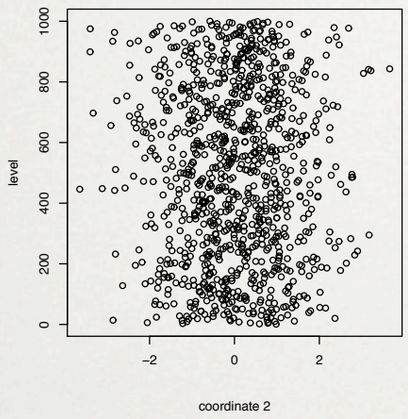


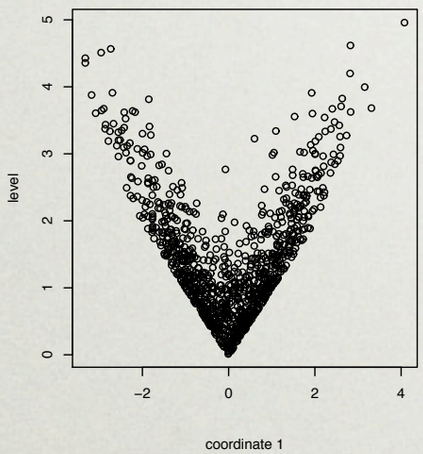
3D RECTANGULAR SPIRAL

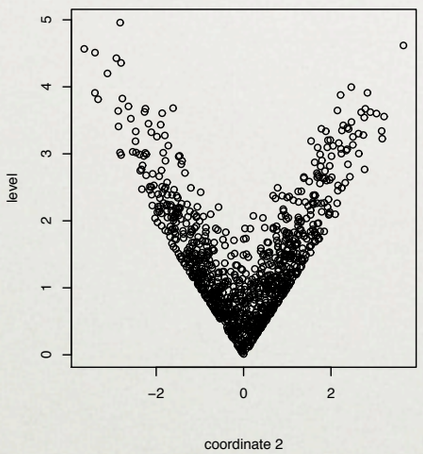


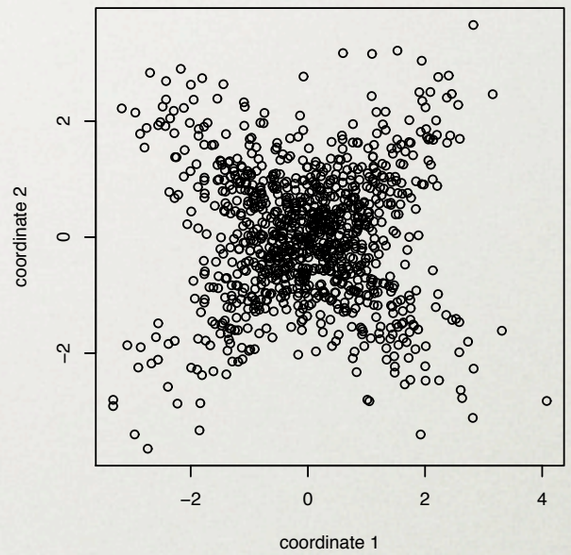
Point Clouds

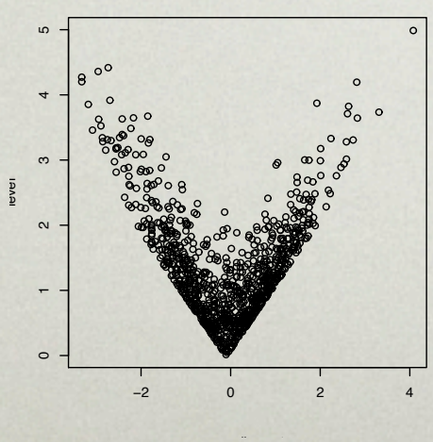


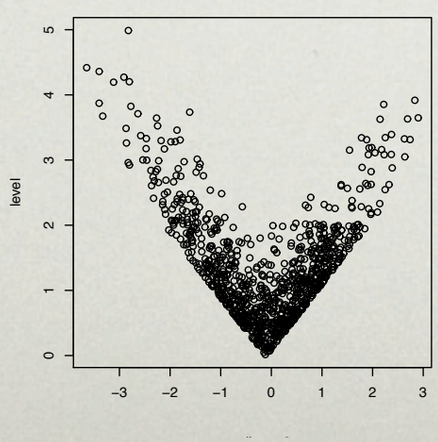


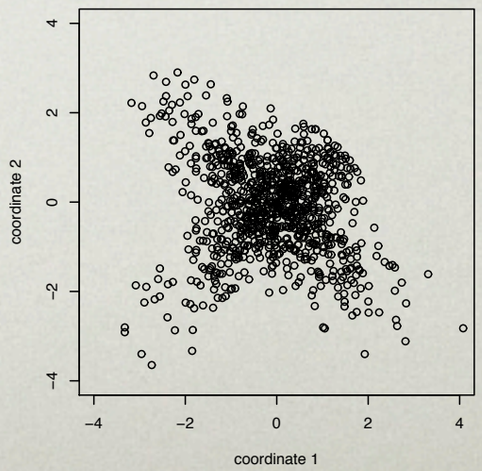




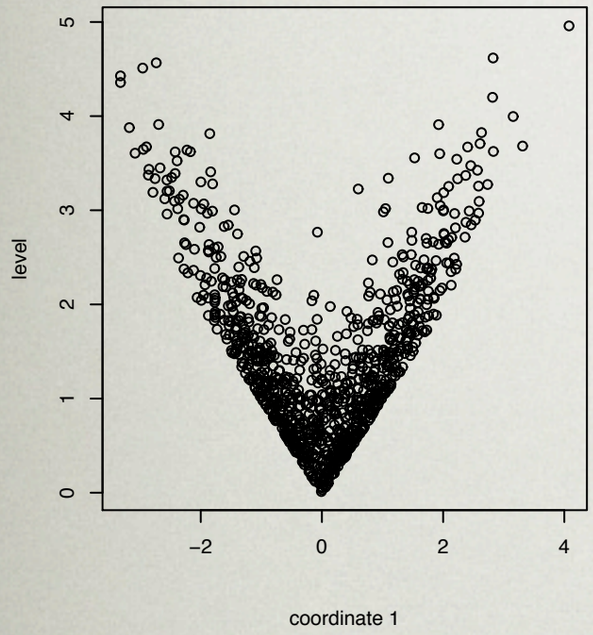


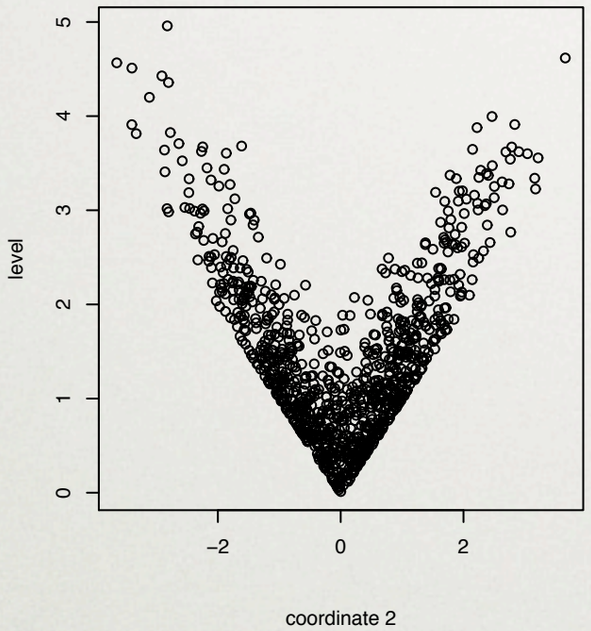


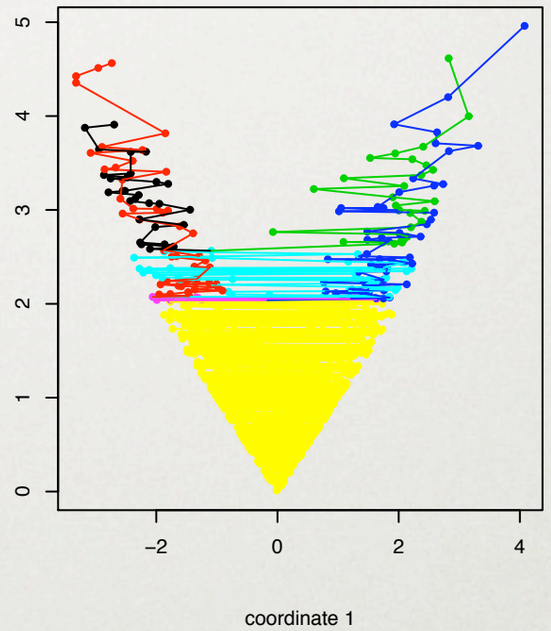


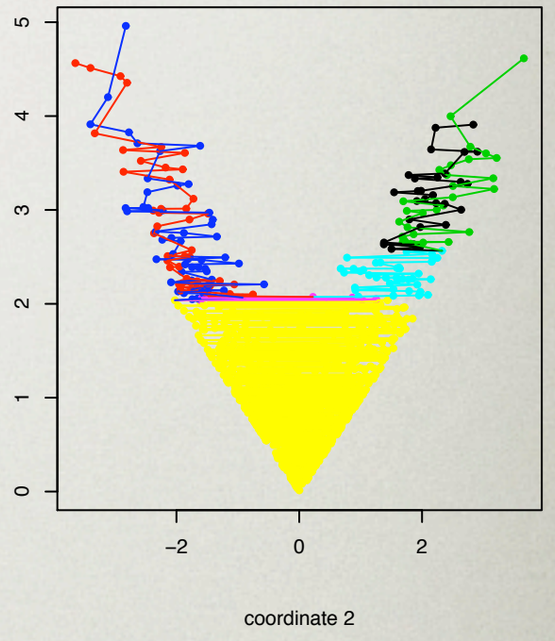


TAILTREE PLOT







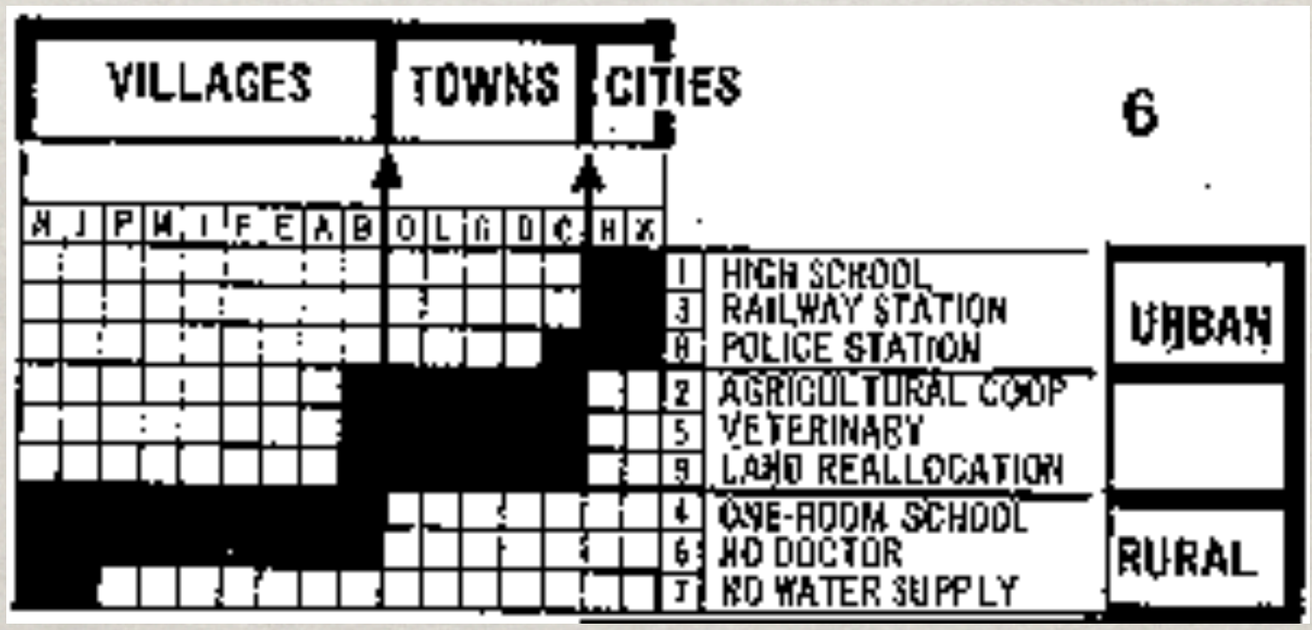


Graphical Matrices

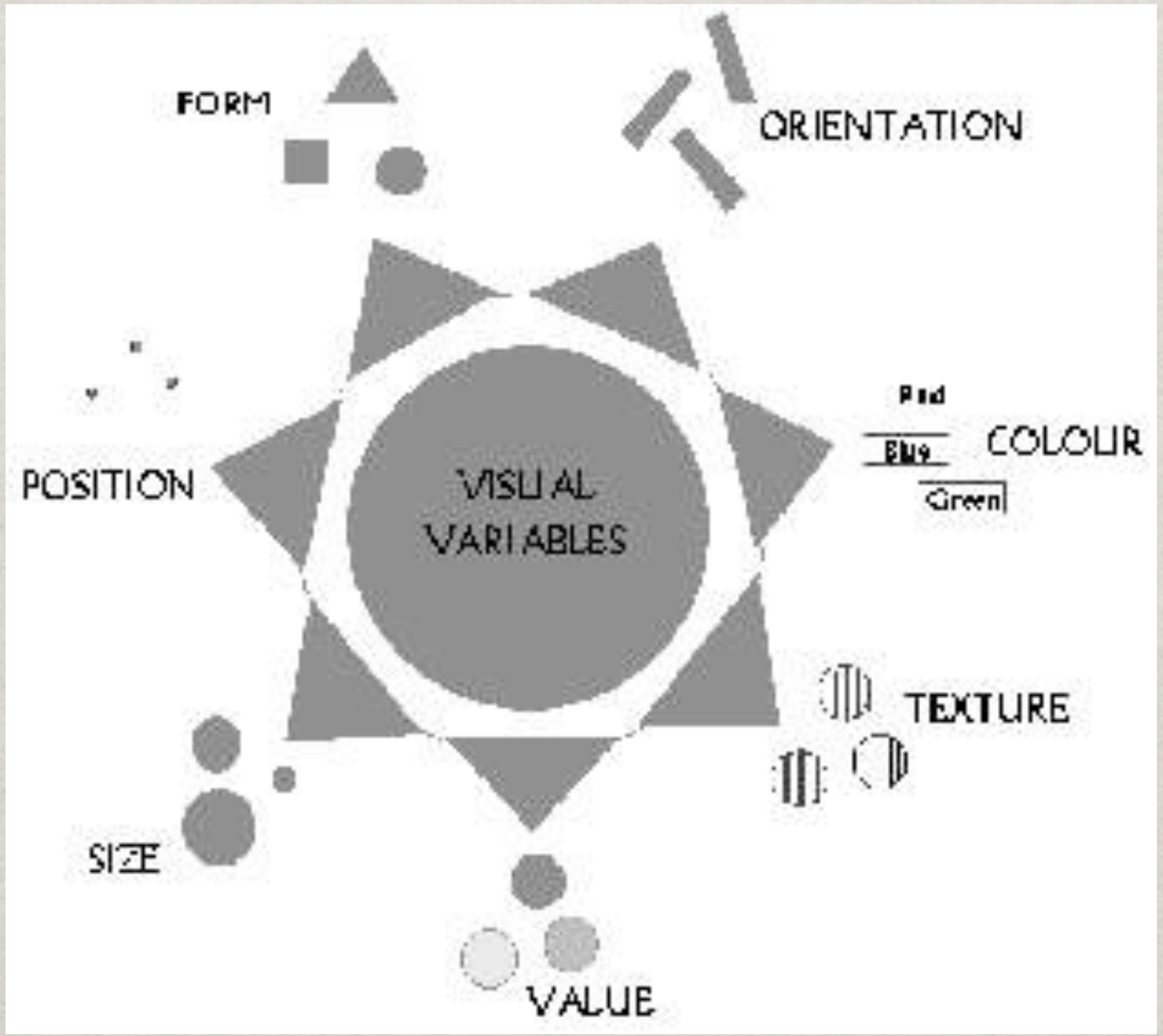
Graphical matrices

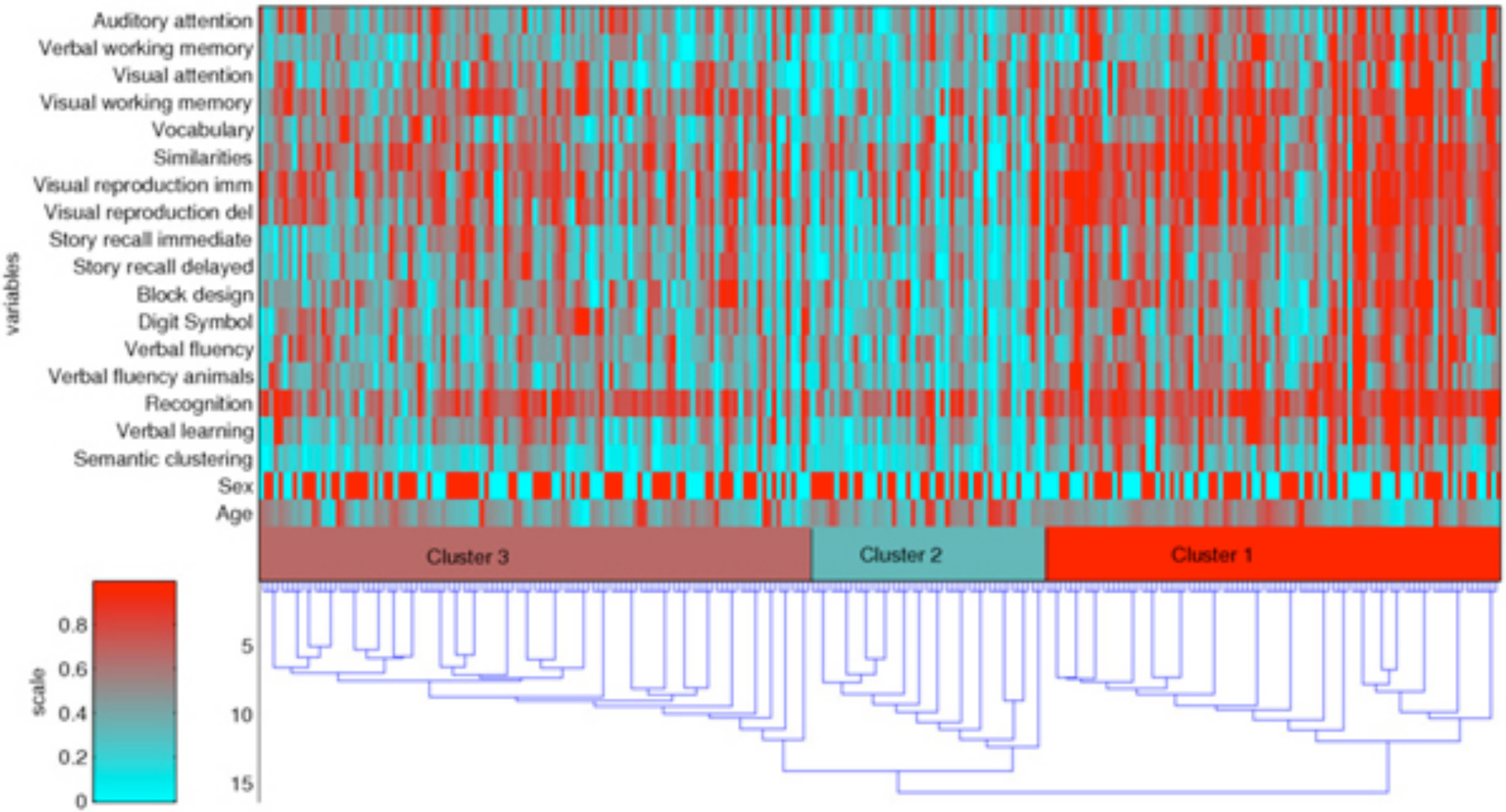
- Bertin (1967), ...
- Graphical matrix is an $n \times d$ matrix of graphical elements
- A data matrix of real numbers will be transformed to a matrix of graphical elements

The reorderable matrix



Semiology of Graphics; 7 visual variables

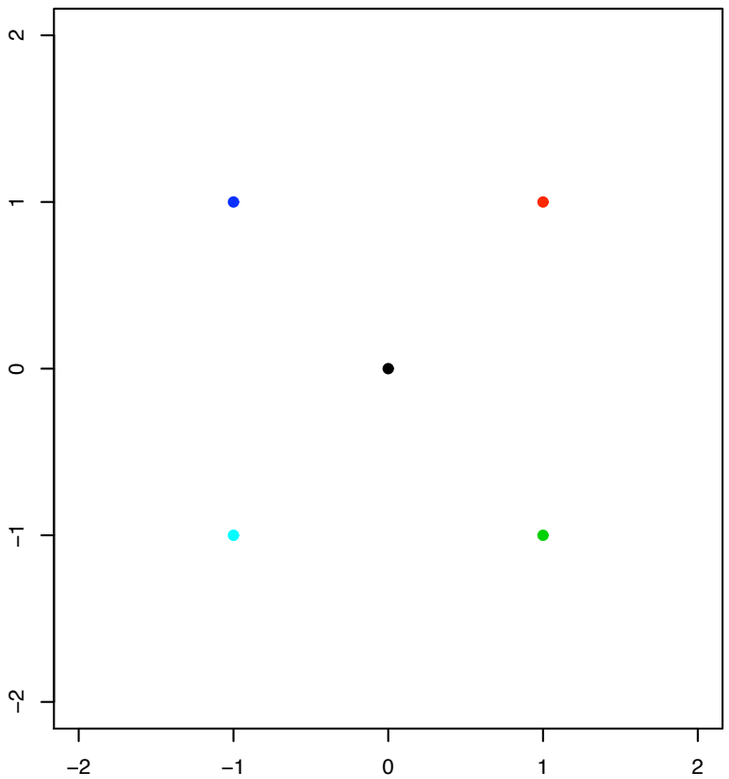


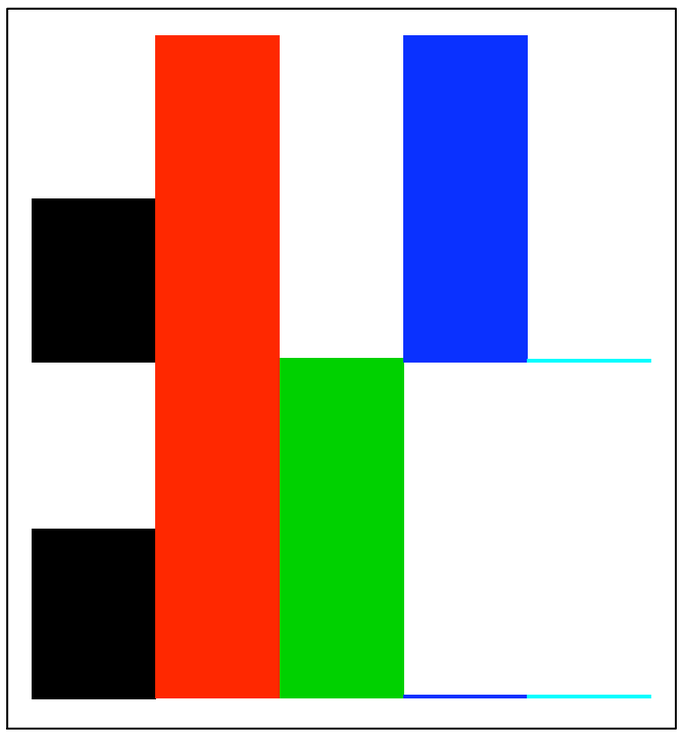


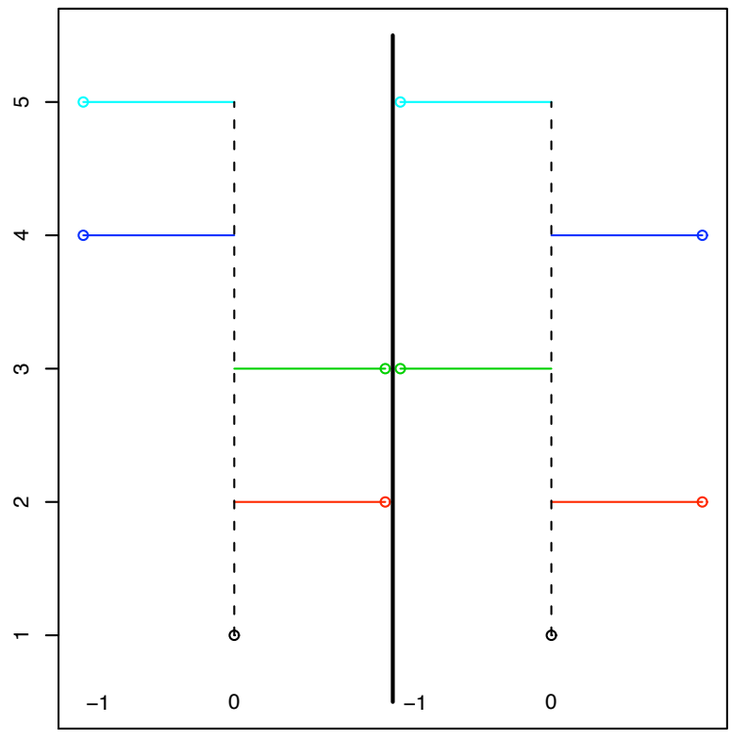
Visualization of clustering result. A visualization of the data and the cluster solution using the data image and the dendrogram. On the last line of the data image clusters 1, 2 and 3 are depicted with different colors. Both the subjects and the neuropsychological tests were ordered by a complete linkage clustering algorithm (see the text).

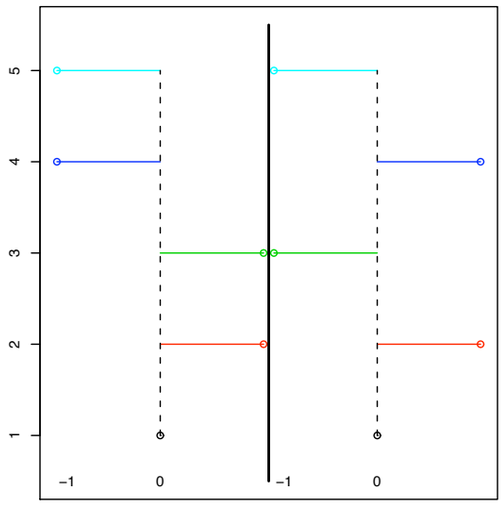
Hoti *et al. BMC Psychiatry* 2004 **4**:20 doi:10.1186/1471-244X-4-20

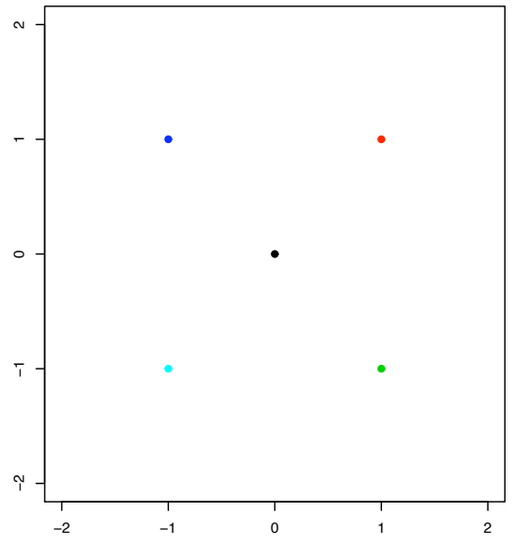
Graphical matrices

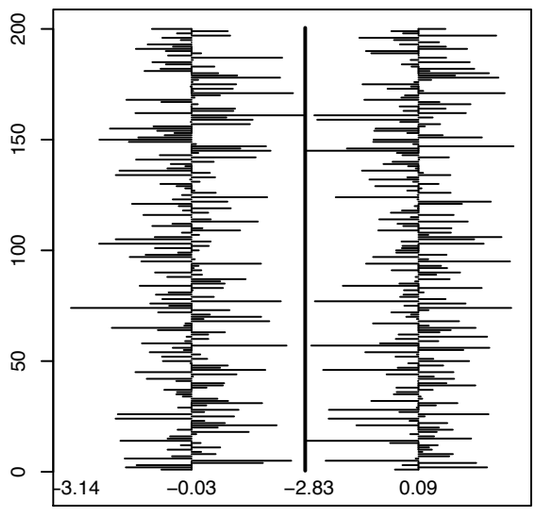


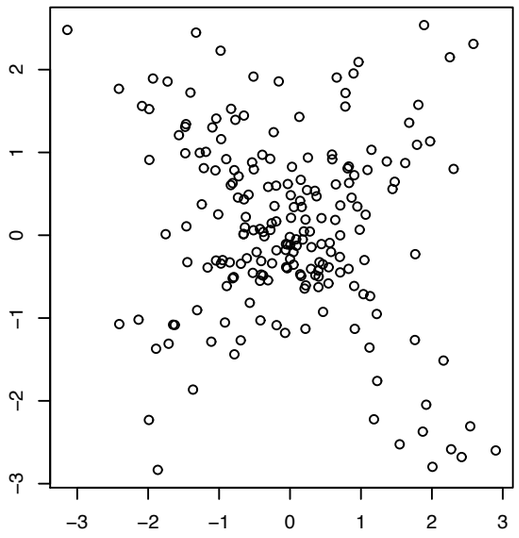


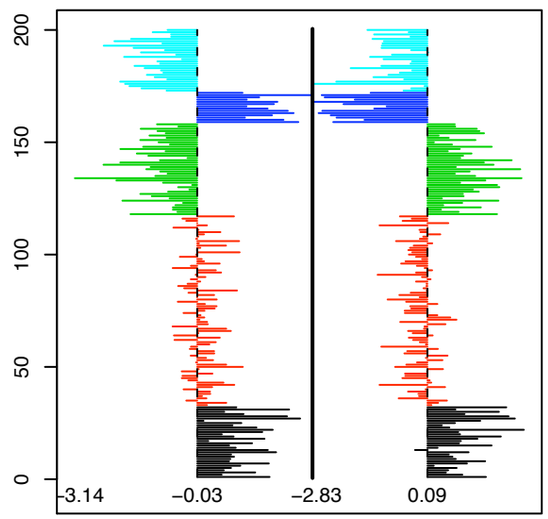


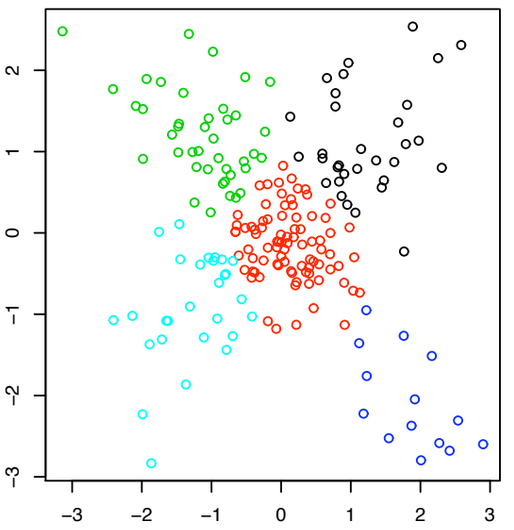

5
4
3
2
1
−1
0
−1
0

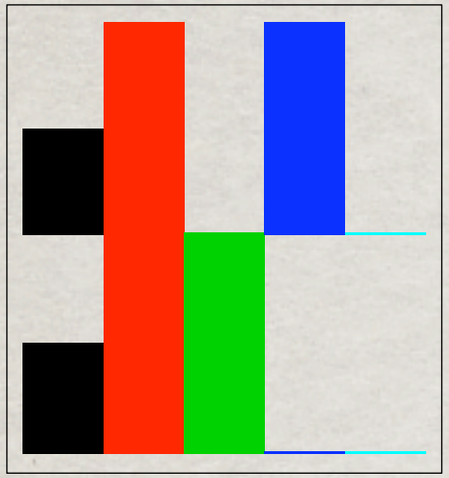

2
1
0
−1
−2
−2
−1
0
1
2

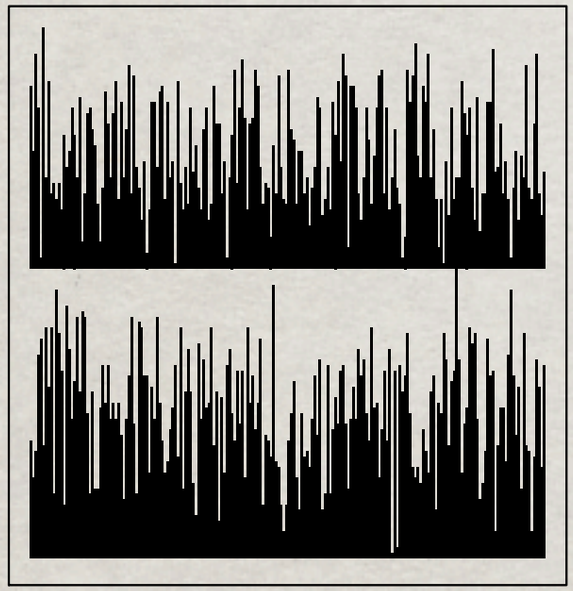

200
150
100
50
0
-3.14
−0.03
−2.83
0.09

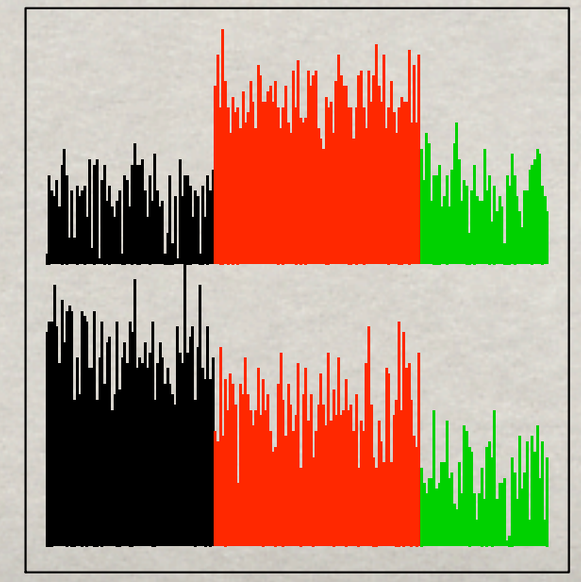

2
1
0
−1
−2
−3
−3
−2
−1
0
1
2
3

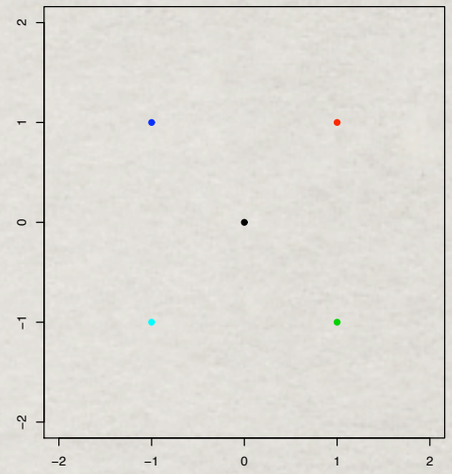

200
150
100
50
0
-3.14
−0.03
−2.83
0.09

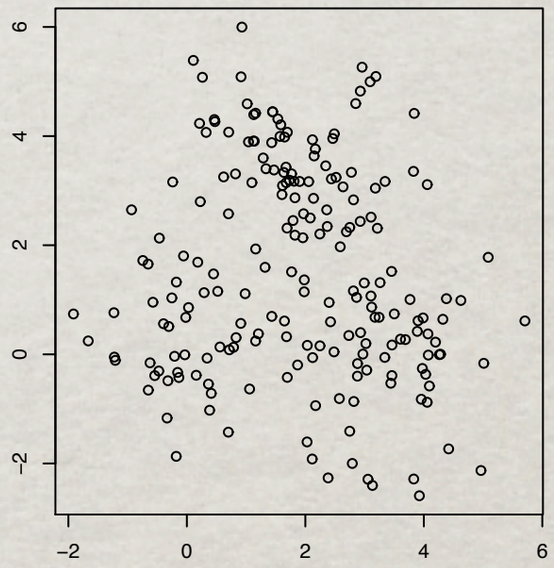

2
1
0
−1
−2
−3
−3
−2
−1
0
1
2
3

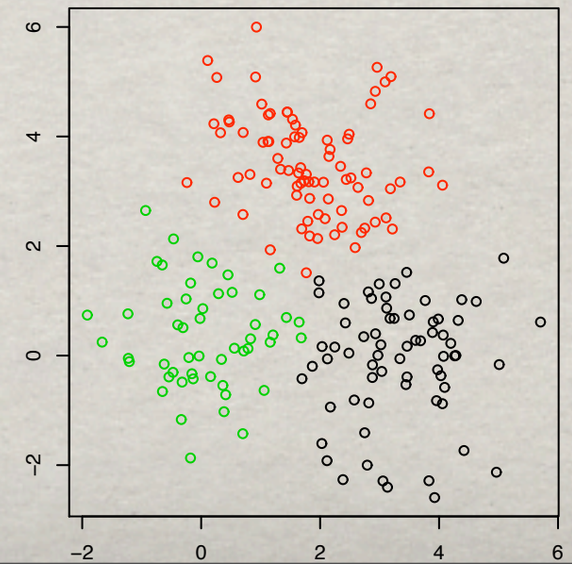






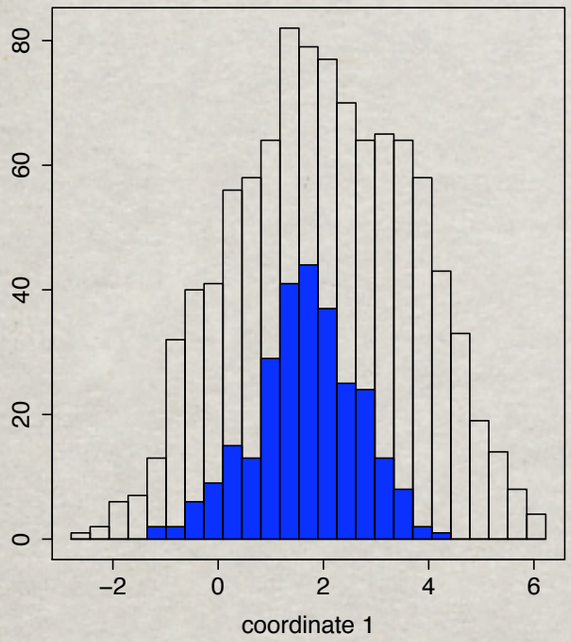

2
1
0
−1
−2
−2
−1
0
1
2

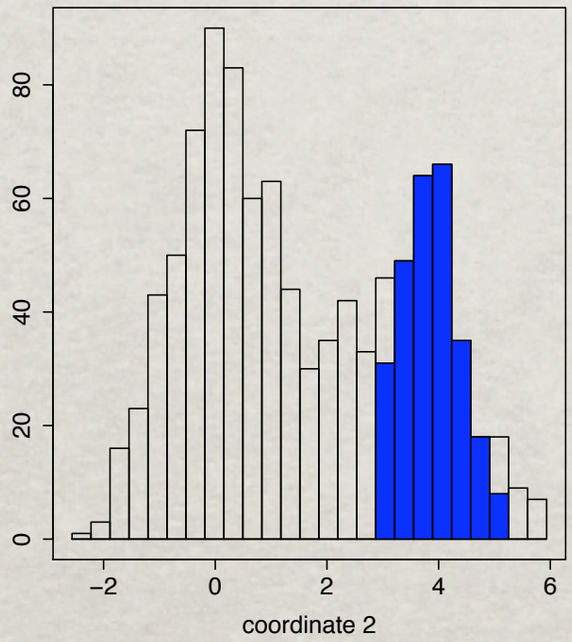

6
4
2
0
−2
−2
0
2
4
6

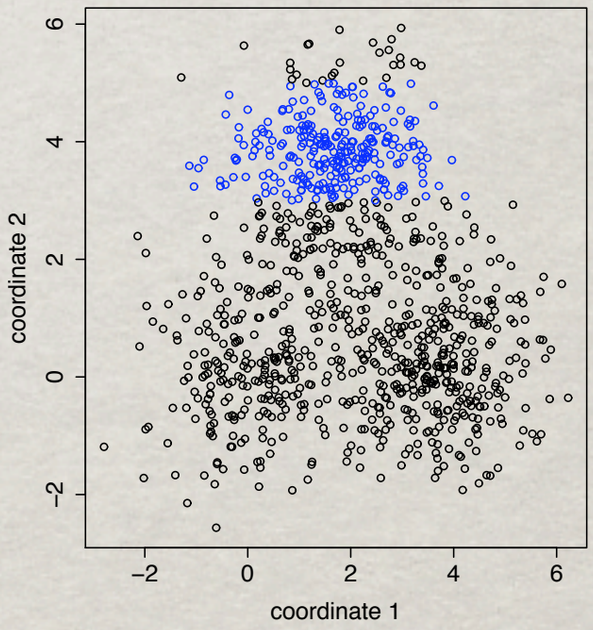

6
4
2
0
−2
−2
0
2
4
6

Linking across dimensions

Histograms


0
20
40
60
80
−2
0
2
4
6
coordinate 1


0
20
40
60
80
−2
0
2
4
6
coordinate 2


−2
0
2
4
6
coordinate 2
−2
0
2
4
6
coordinate 1

VISUALIZATION OF FUNCTIONS

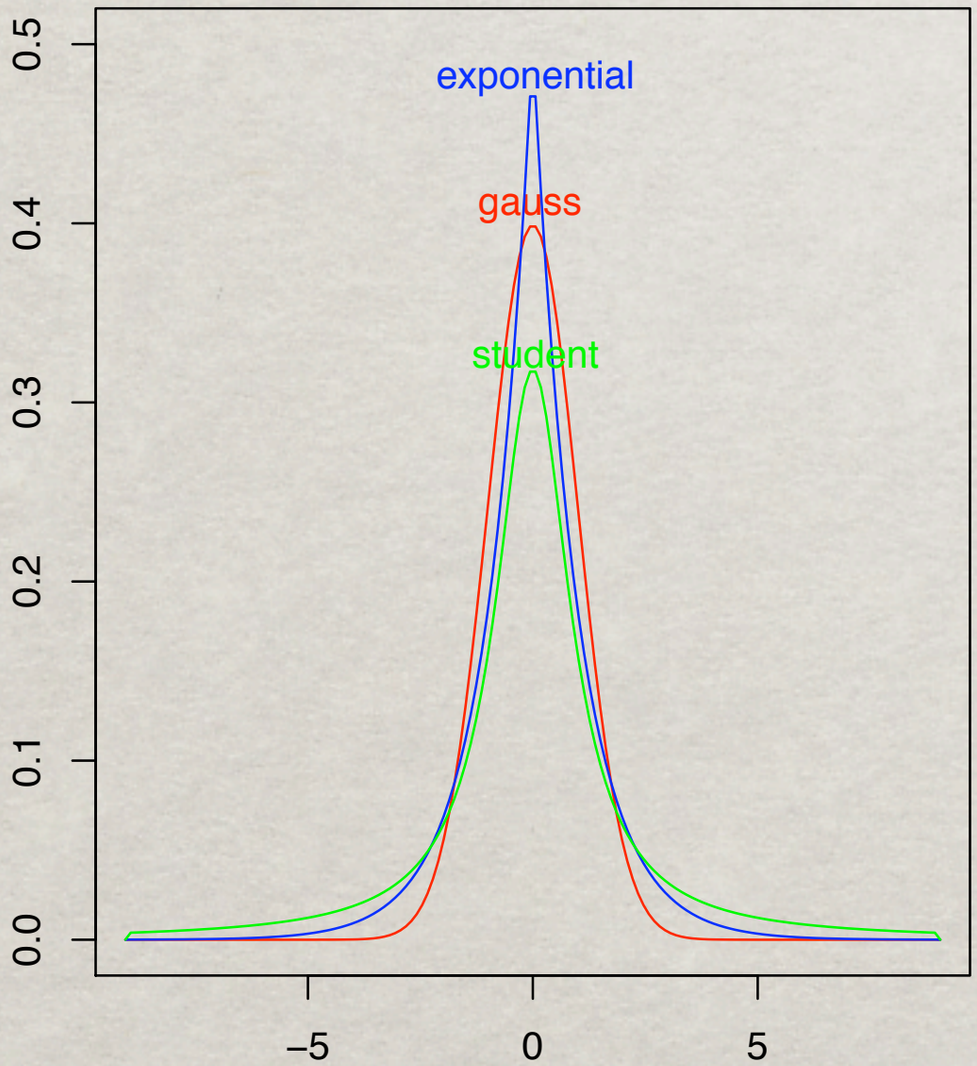
1D Functions

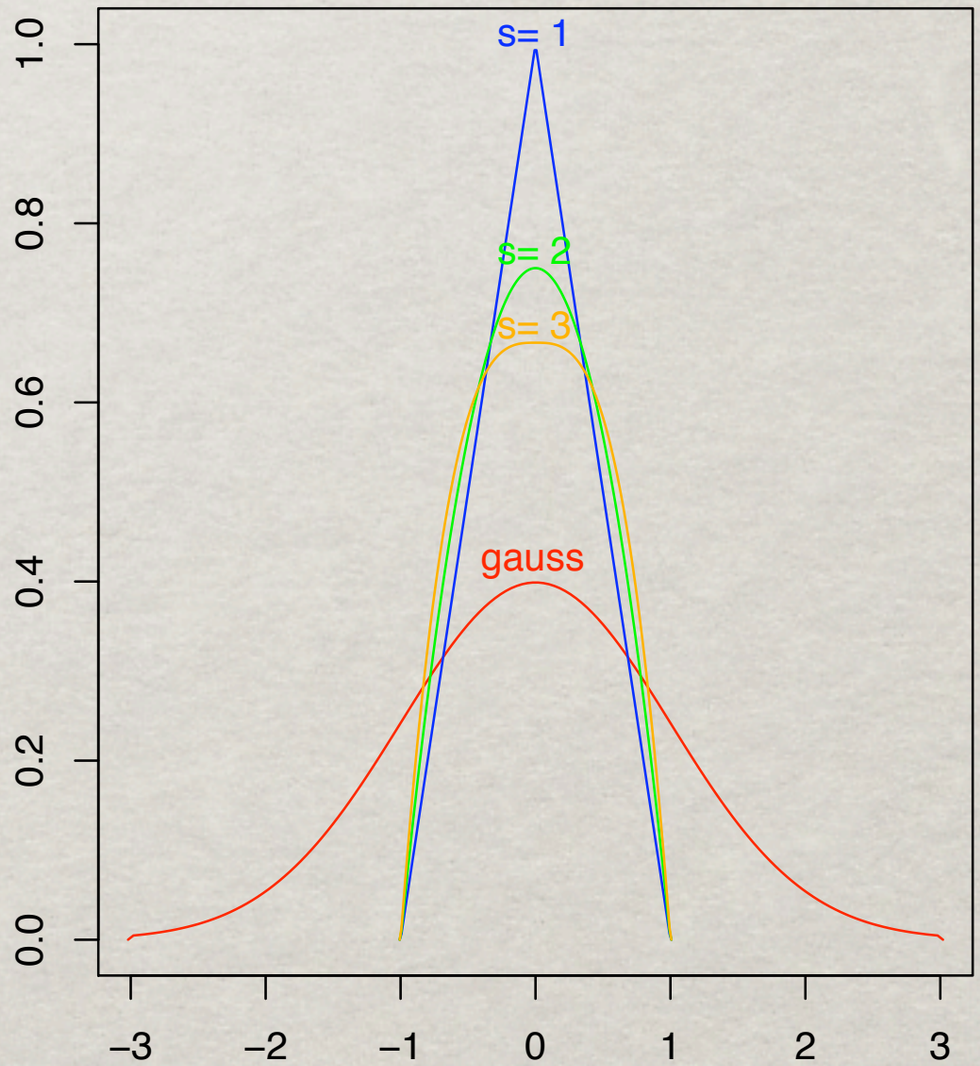
Graph of 1D functions

Graph. The graph of the function $f : \mathbf{R} \to \mathbf{R}$ is the set of points

$$\{(t, f(t)) : t \in \mathbf{R}\}.$$

Graph of 1D functions





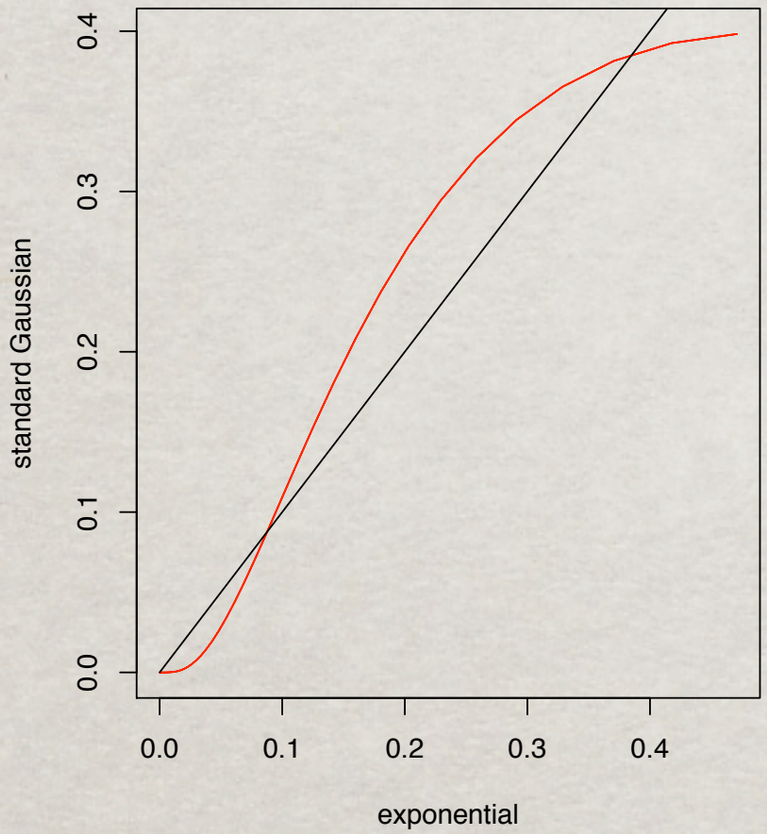
FF-PLOT

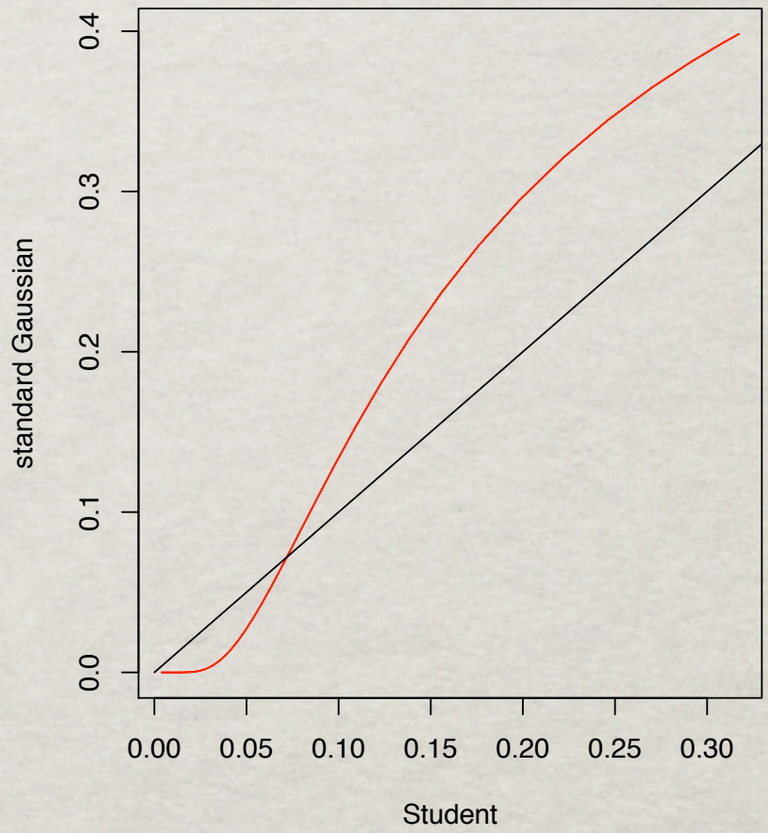
ff-plot. An other way to visualize function f is to plot the set of points

$$\{(f(t), \phi(t)) : t \in \mathbf{R}\},$$

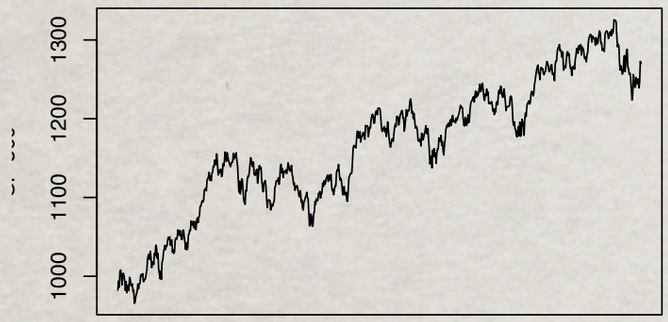
where $\phi : \mathbf{R} \to \mathbf{R}$ is some reference function.

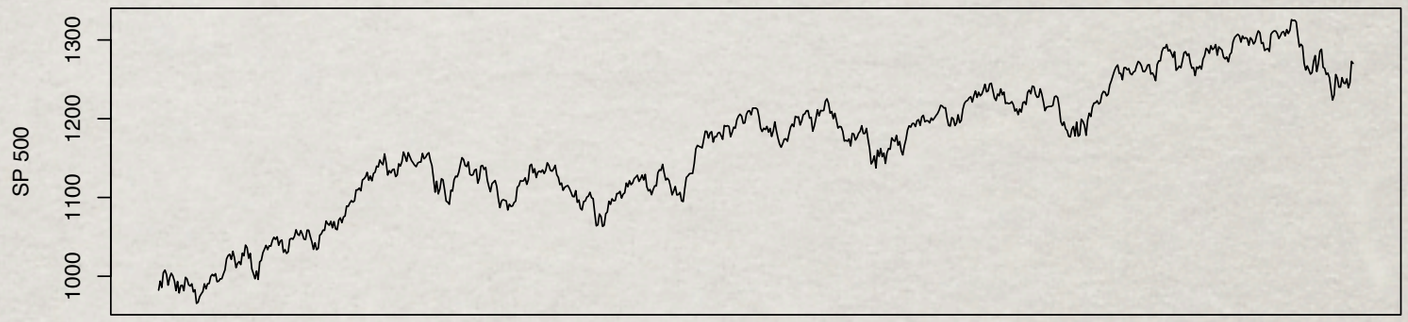
FF-PLOT

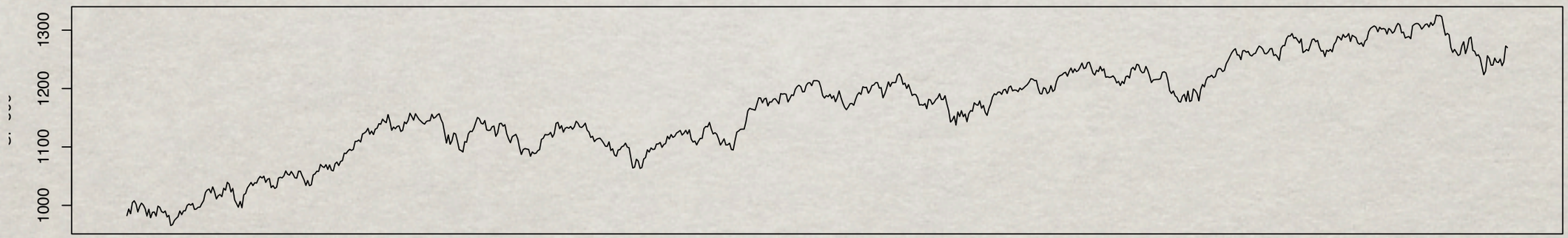

standard Gaussian
0.0
0.1
0.2
0.3
0.4
0.0
0.1
0.2
0.3
0.4
exponential


standard Gaussian
0.0
0.1
0.2
0.3
0.4
0.00
0.05
0.10
0.15
0.20
0.25
0.30
Student

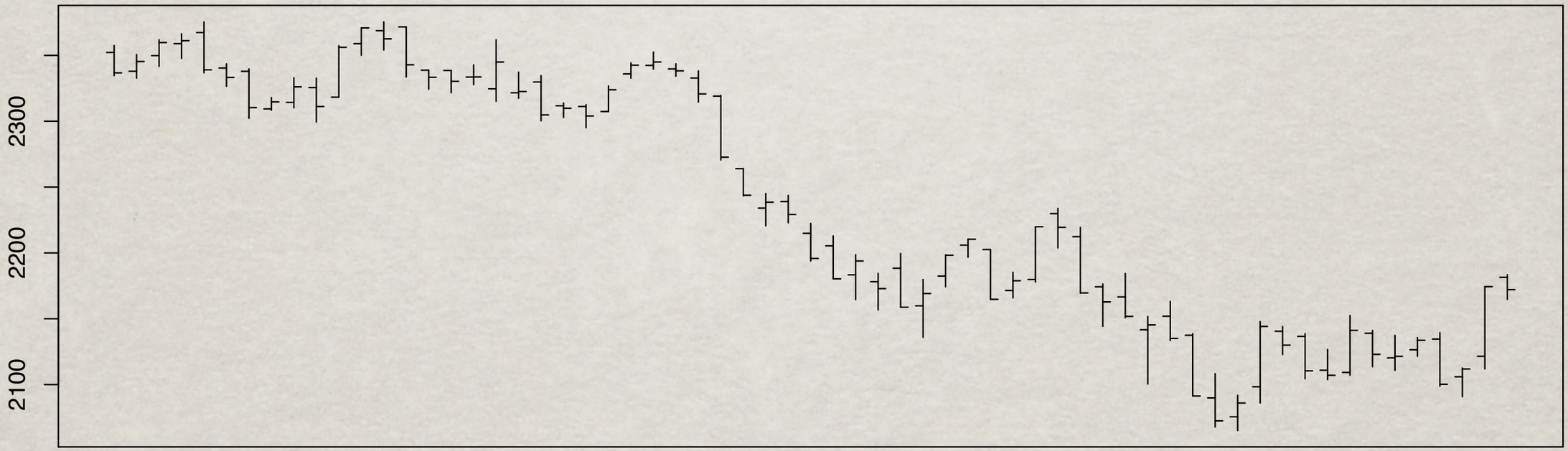
Aspect Ratio



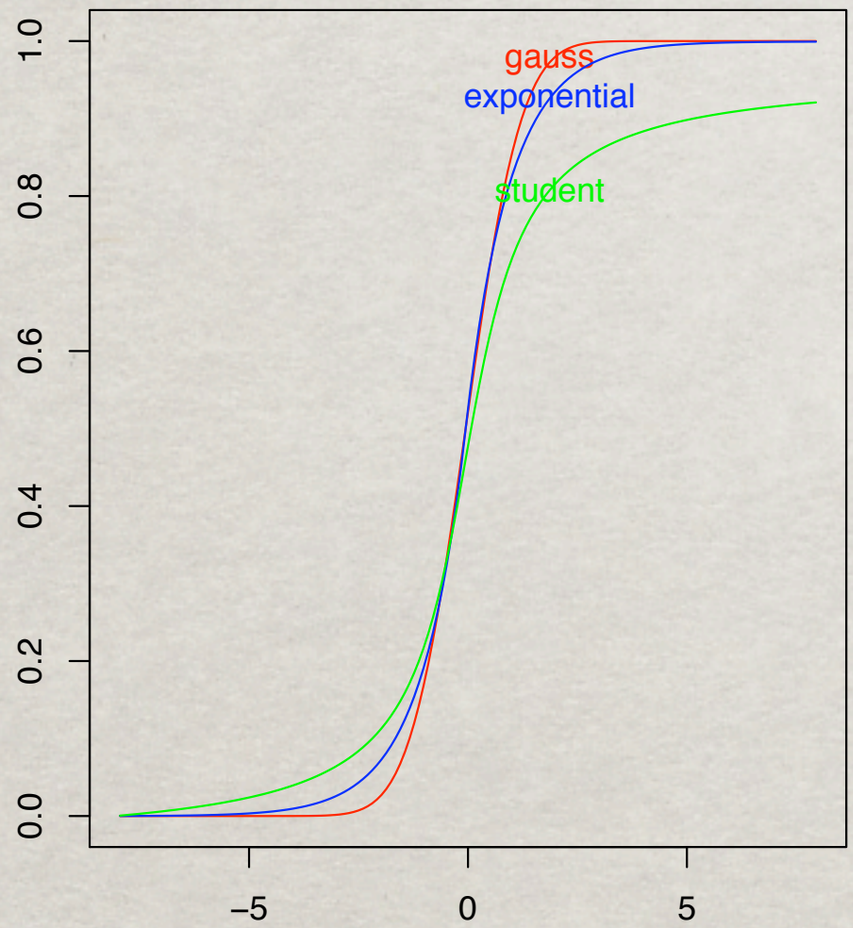


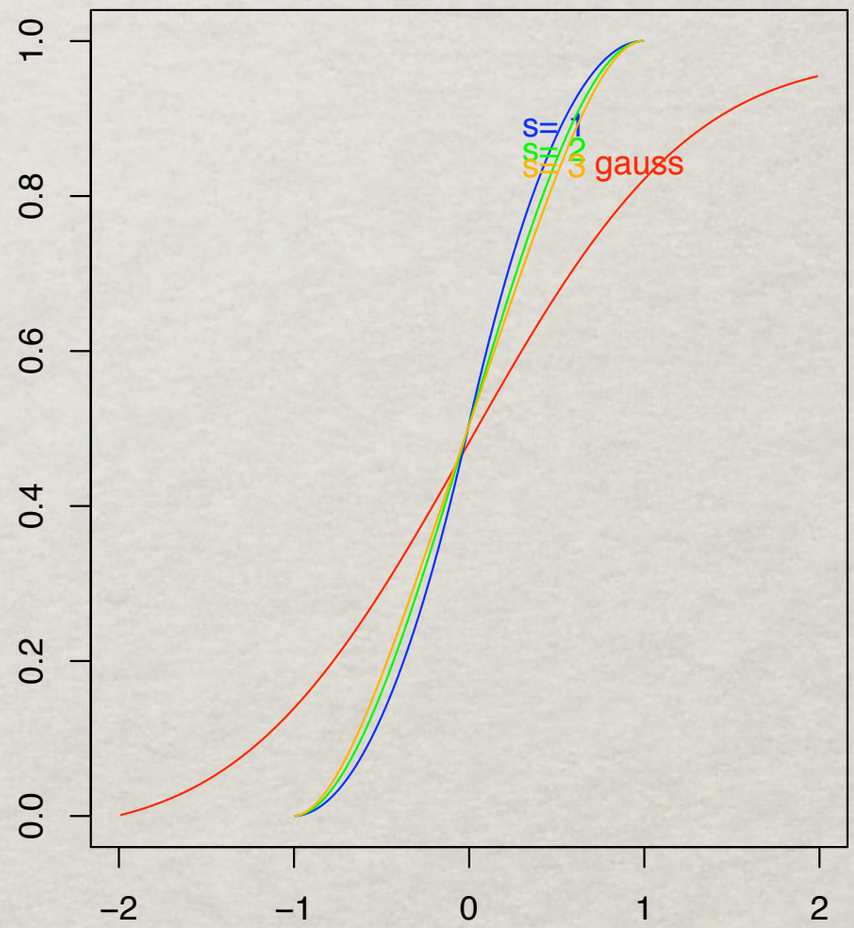


Irregular functions



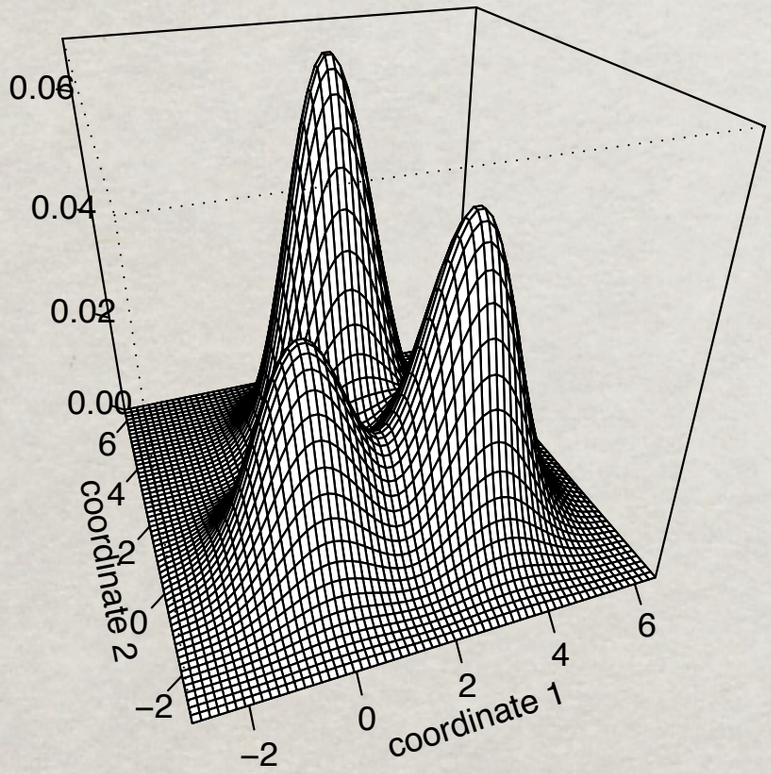
Distribution functions



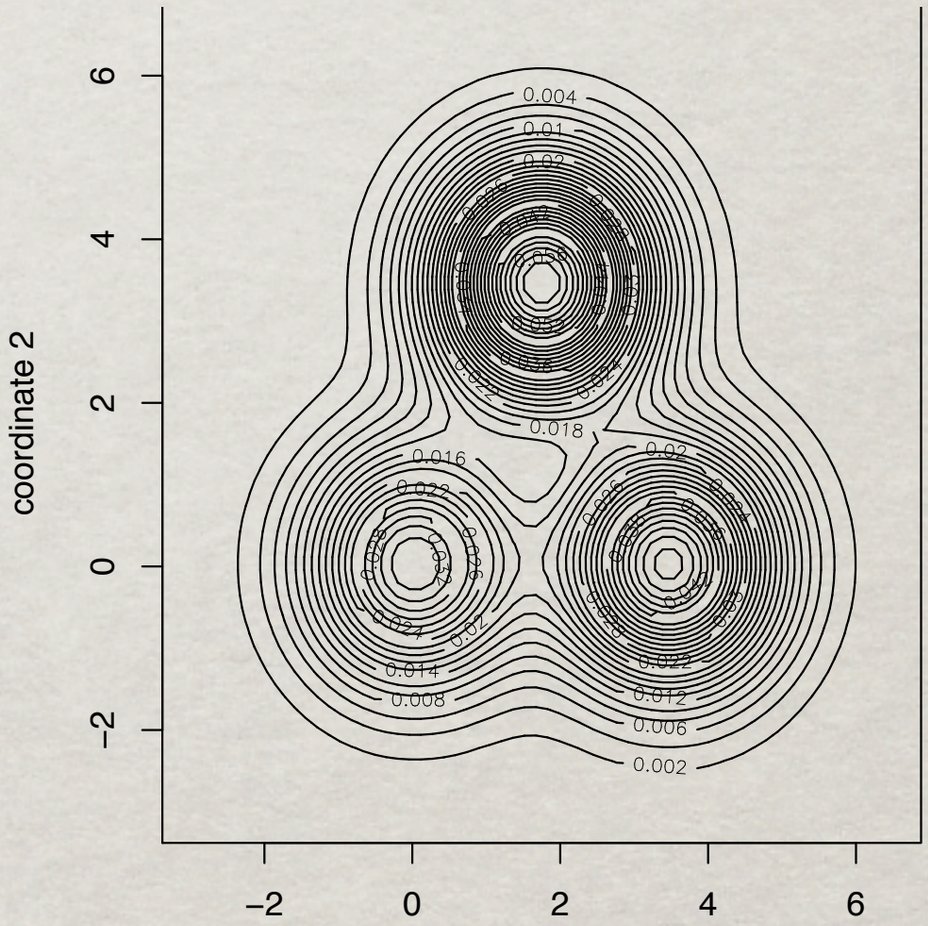


2D functions

Perspective and Contour plot



perspective plot



coordinate 1
contour plot

Perspective plots

- wireframe plot, with hidden line removal
- “surface plot”, “mesh plot”, “wire mesh surface”

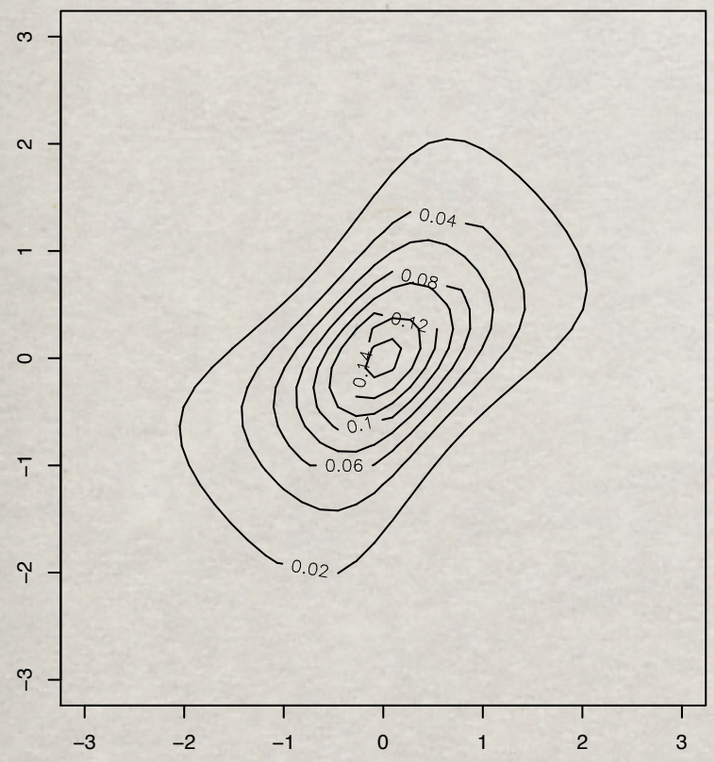
Perspective plots

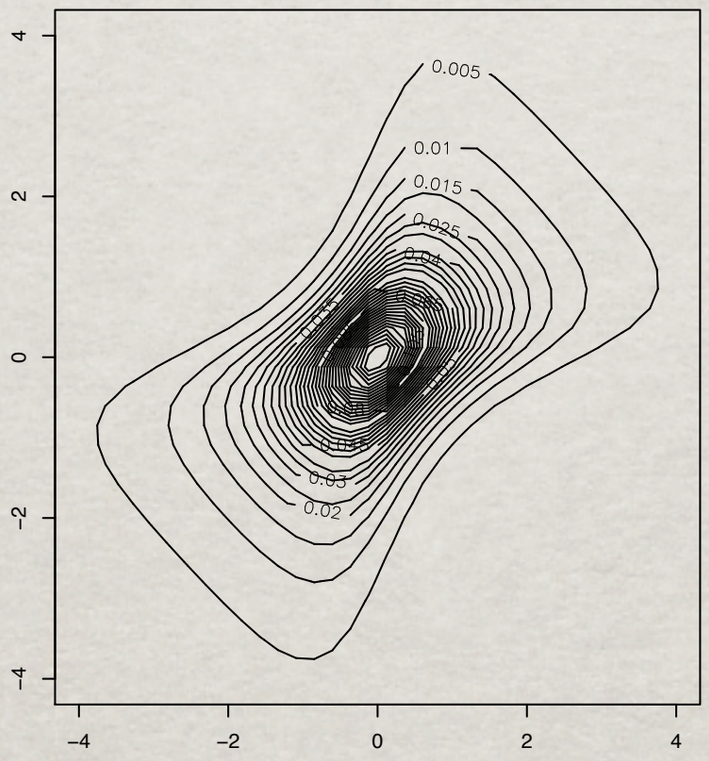
Cleveland (1993*b*), page 249, lists the following choices to be made in wireframe rendering: grid density (the grid density cannot be too large since then the lines connecting the points merge and the perception of the depth is lost), the orientation of the surface, viewing distance, ratios of the lengths of the three dimensions of the plot, overall size of the display. One may also use an orthogonal view, instead of rendering the surface in perspective. In an orthogonal view parallel lines moving from foreground to background stay a constant distance apart.

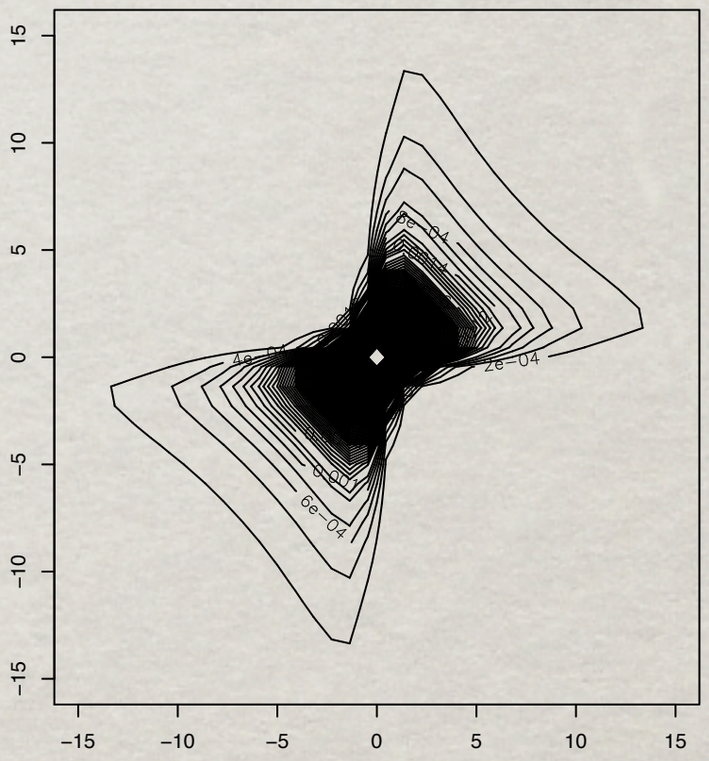
Shaded Surface

An other way to make a perspective plot is to use a shaded surface. Cleveland (1993*b*), page 268, lists the following choices to be made in shading: reflective properties of the surface (matte or shiny finish), color of the surface, number, color, intensity, and location of point light sources, color and intensity of ambient light. Foley, van Dam, Feiner and Hughes (1997) and Watt (2000) give more details on computer graphics. Rock (1984) and Marr (1992) discuss principles of vision and perception.

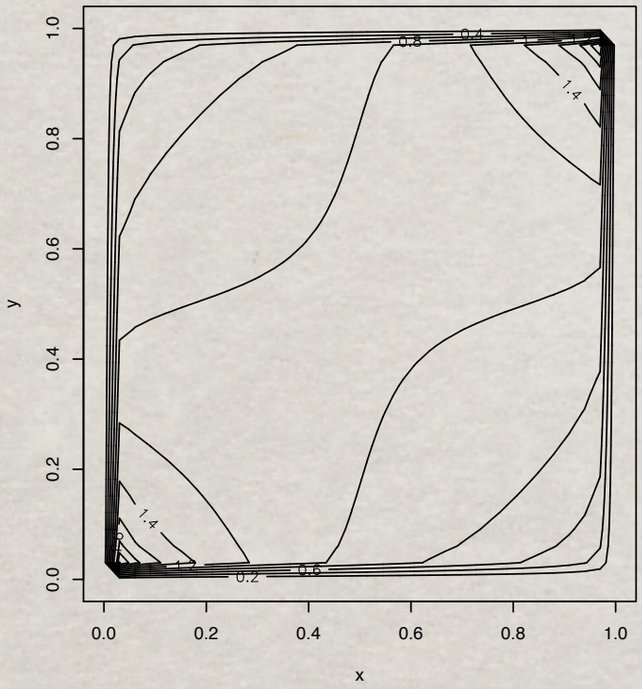
Contour plots

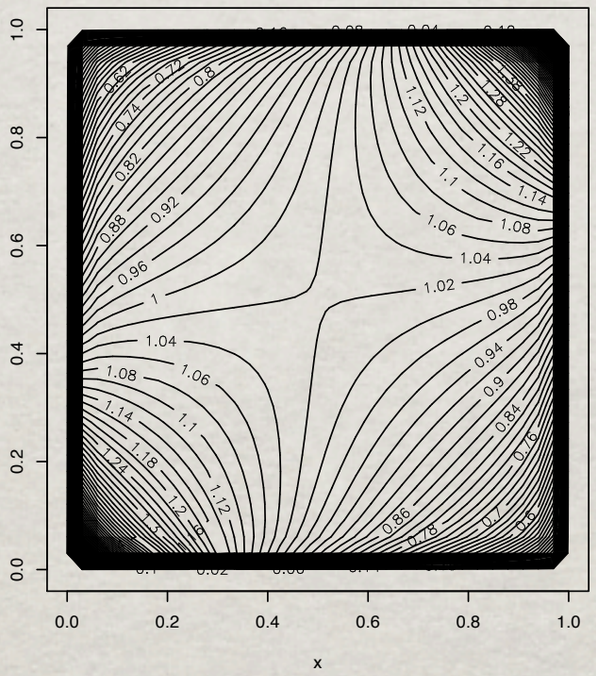


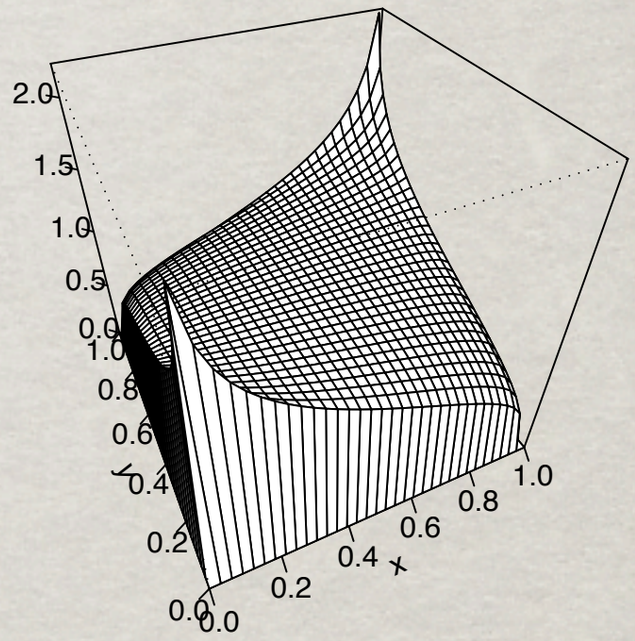




Perspective vs. Contour







Multivariate functions

Slices

slice of function f is

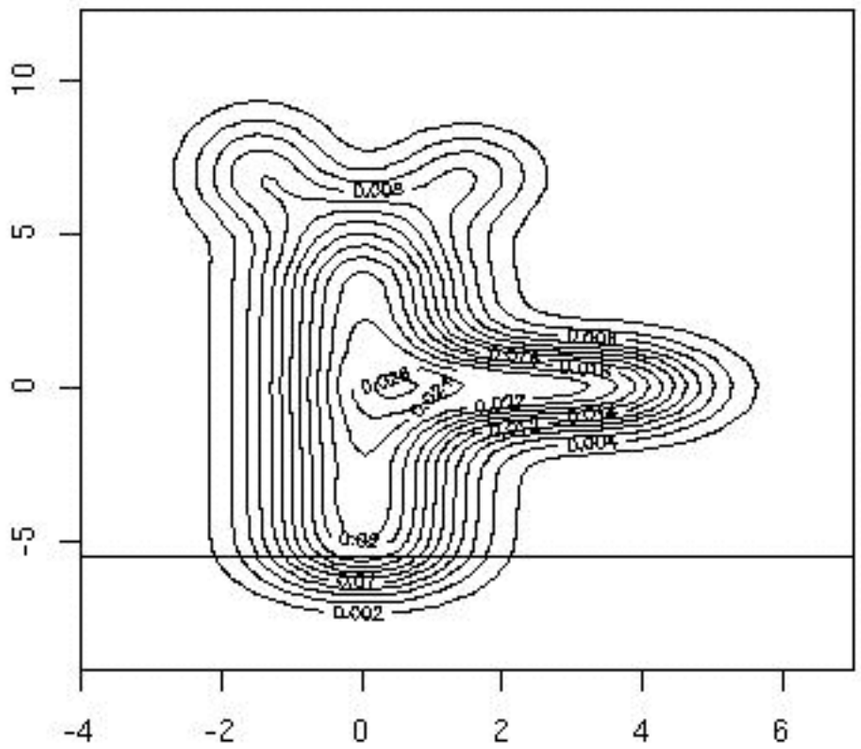
$$f_{X_1,X_2,X_3=a_3,\ldots,X_d=a_d}(x_1,x_2) = f(x_1,x_2,a_3,\ldots,a_d)$$

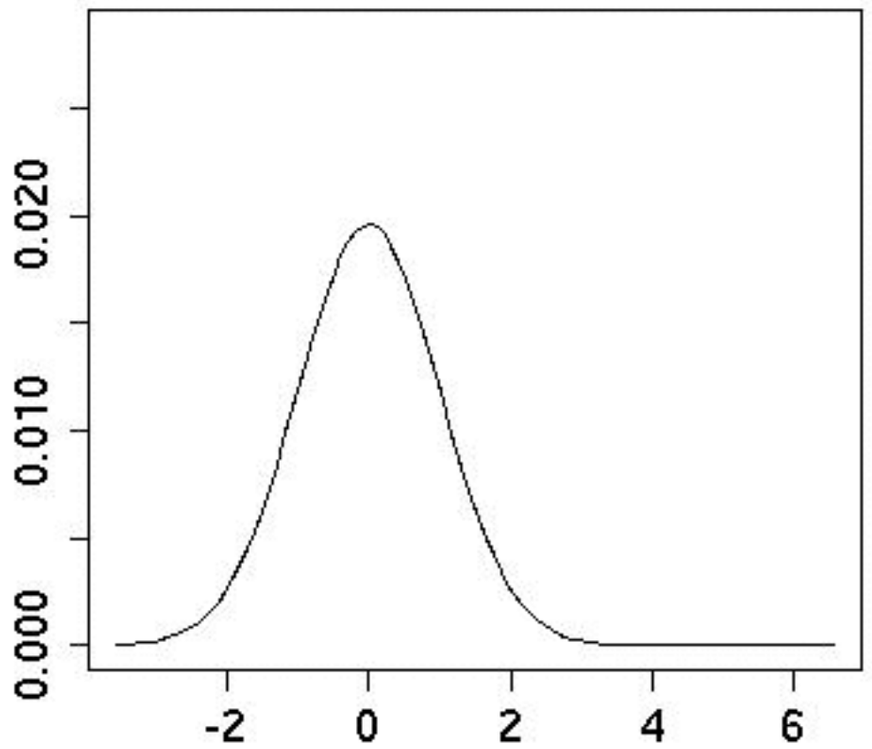
where we have fixed variables $3-d$ to take the values $a_3,\ldots,a_d \in \mathbf{R}$

even more generally, we may define slice $g : \mathbf{R}^{d'} \to \mathbf{R}$ by

$$g(x_1,\ldots,x_{d'}) = f(\phi(x_1,\ldots,x_{d'})),$$

where $\phi : \mathbf{R}^{d'} \to \mathbf{R}^d$, with $1 \leq d' < d$. For example, in the slice of equation


10
5
0
-5
-4
-2
0
2
4
6


0.020
0.010
0.000
-2
0
2
4
6

Marginal Density

- "projection" of densities
- $$f_{X_1,X_2}(x_1,x_2) = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} f(x_1,x_2,x_3,\ldots,x_d)\,dx_3 \cdots dx_d.$$

Radon transform

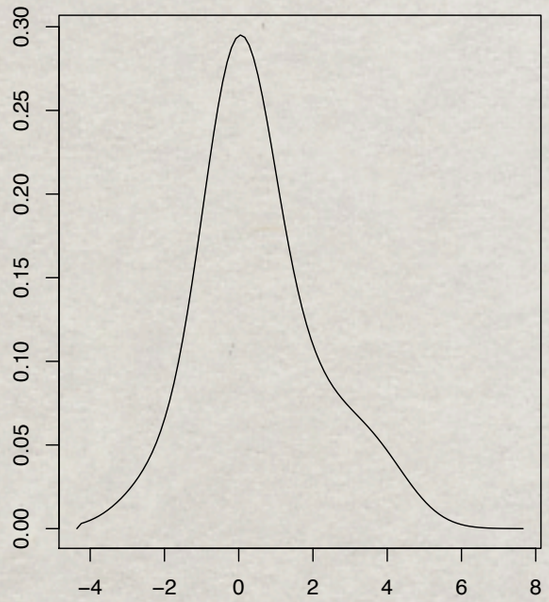
- "projection" of functions

- hyperplanes, $1 \leq d' < d$. Define the d'-dimensional Radon transform as the function

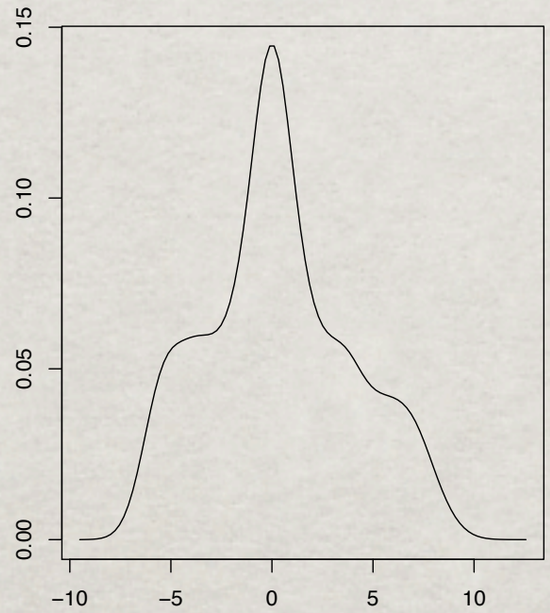
$$(Rf)(P) = \int_P f, \qquad P \in \mathcal{P},$$

where $\mathcal{P}$ is the set of d'-dimensional hyperplanes, and the integration is with

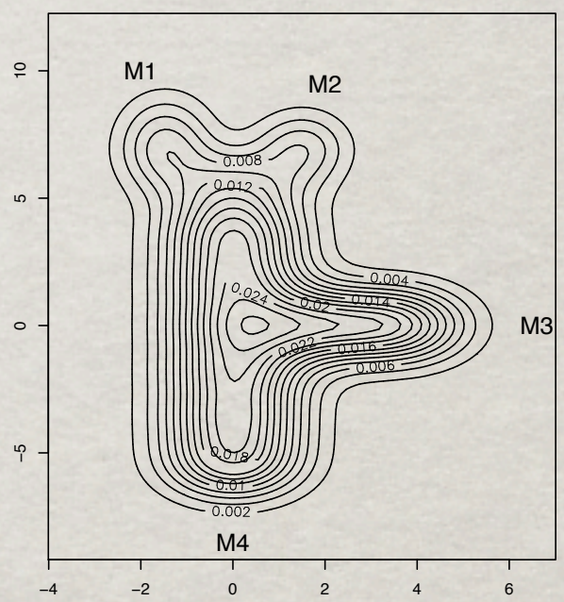
Examples of marginals



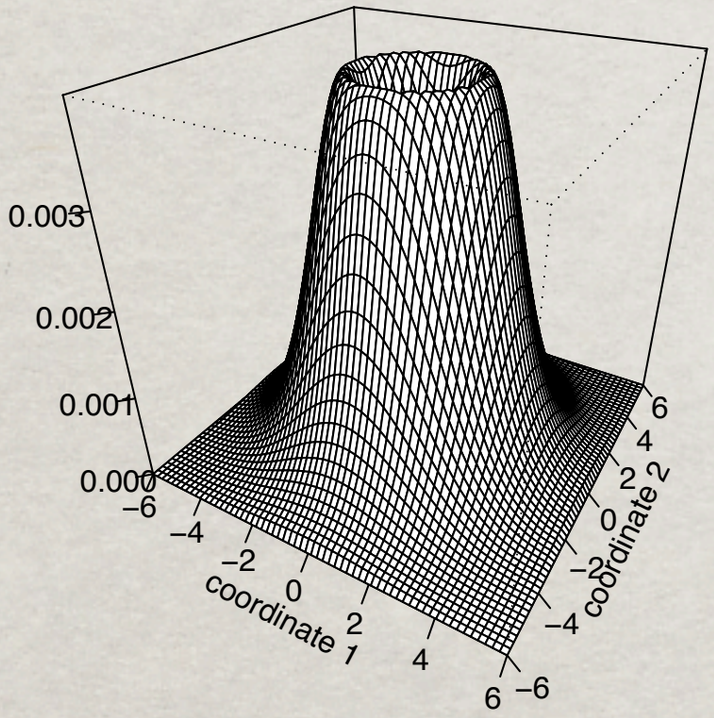
a) projection to x−axis



b) projection to y−axis



An other example



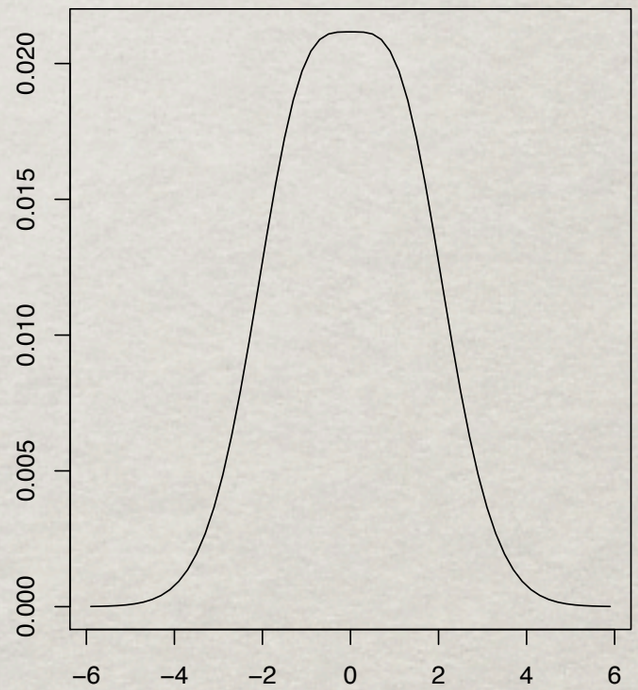
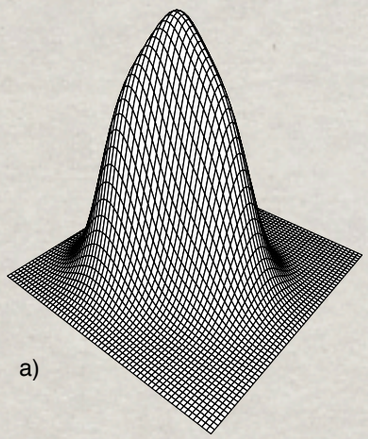
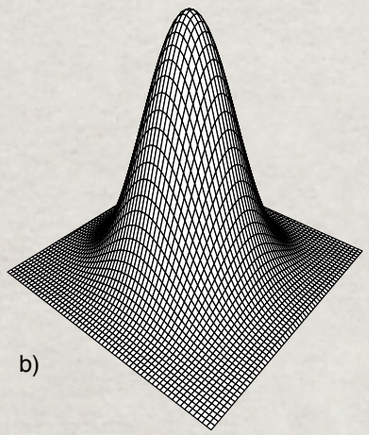
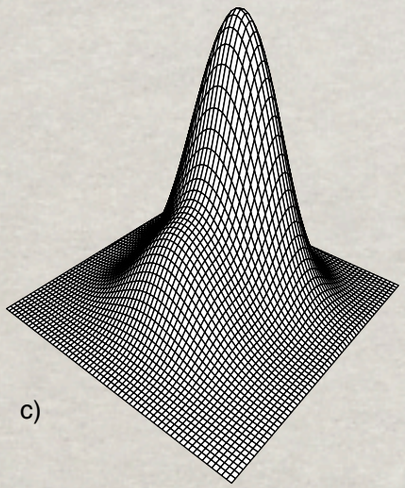
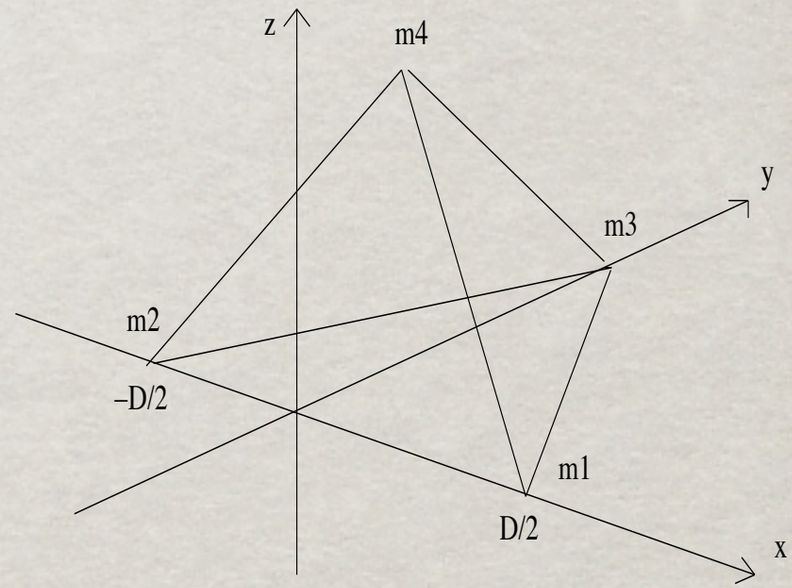


Figure 3.18: Marginal densities of a mixture of 4 standard Gaussian densities in $\mathbf{R}^3$; a) x-y plane, D=3; b) x-z plane, D=3; c) y-z plane, D=3; d) x-y plane, D=4.









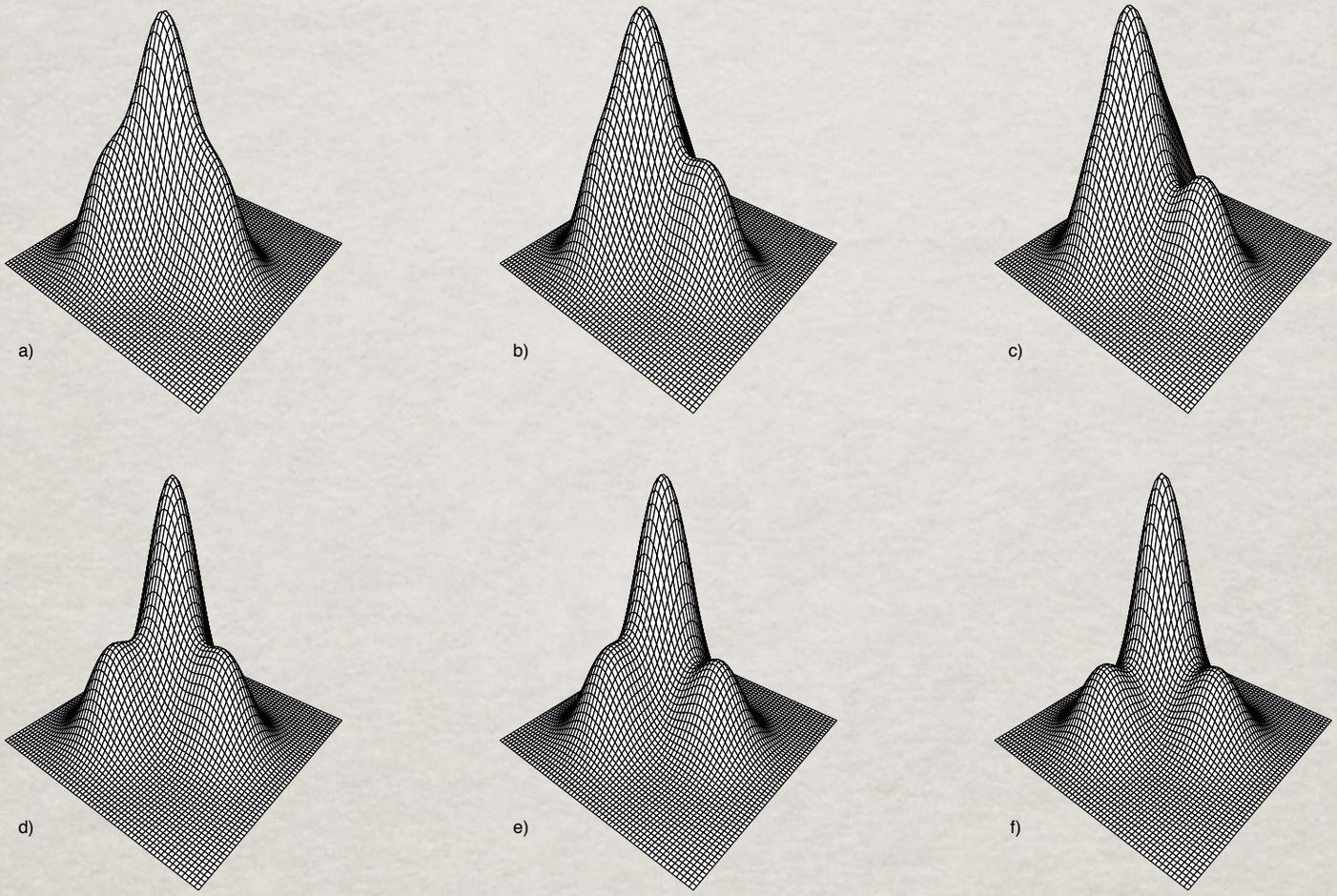


Figure 3.19: Marginal densities of a mixture of 5 standard Gaussian densities in $\mathbf{R}^4$, when $D = 4$; a) x-y plane; b) x-z plane; c) x-u plane; d) y-z plane; e) y-u plane; f) z-u plane.

Shape isomorphic transforms

Level set tree

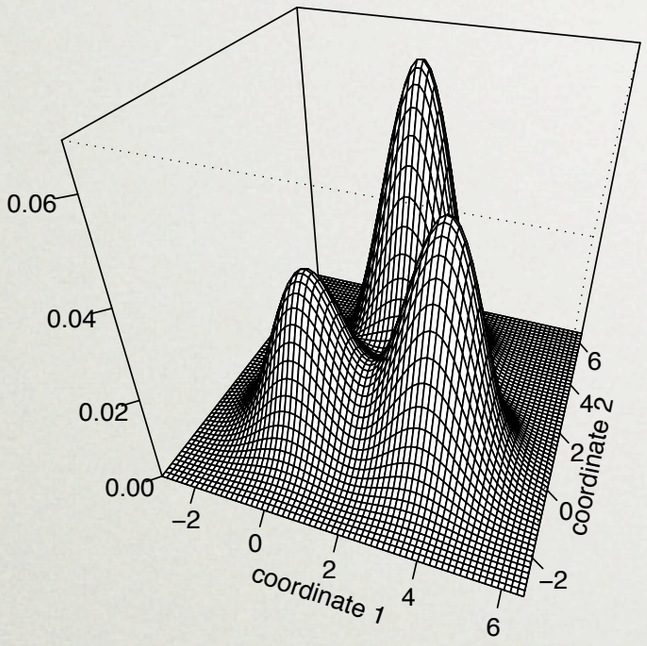


Fig. 1. A perspective plot of a 3-modal function.

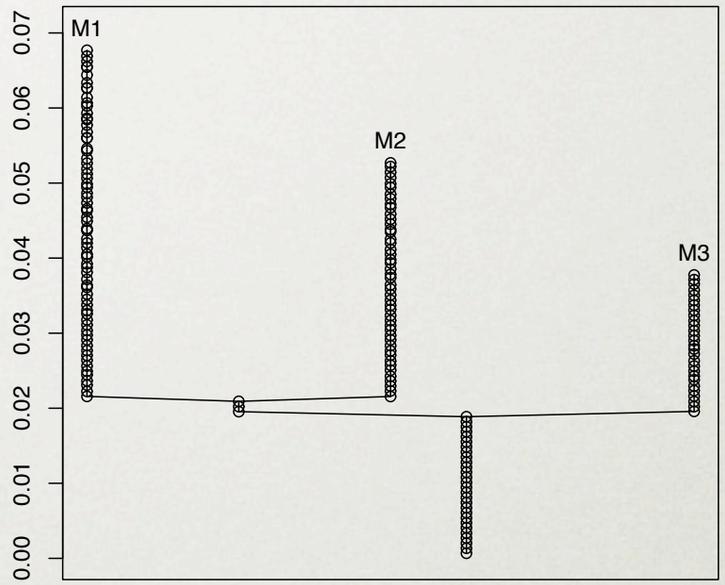


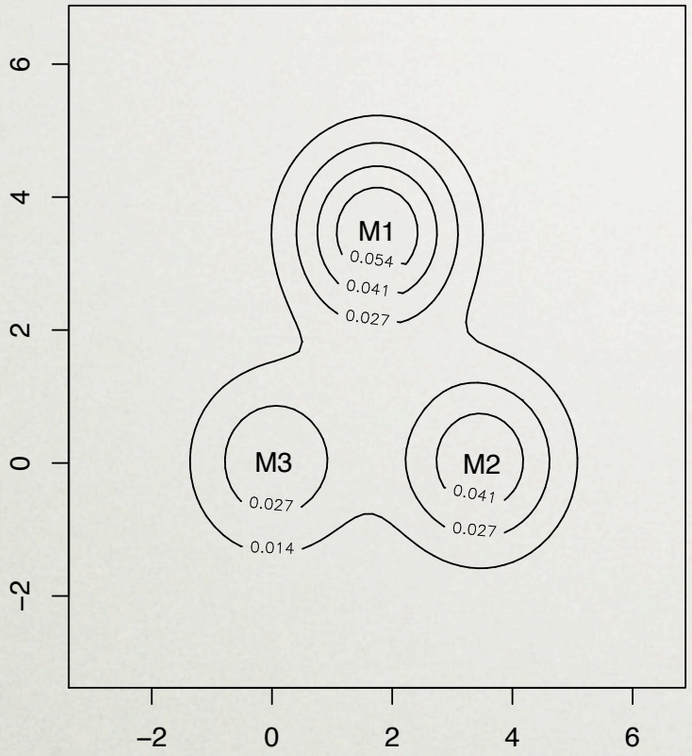
Fig. 2. A level set tree of the 3-modal function.

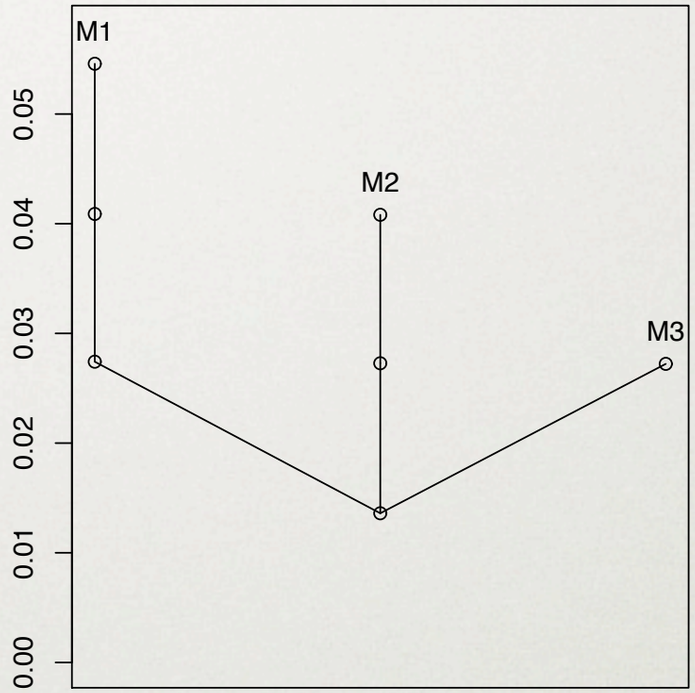
Level set

A *level set* of a function is the set of those points where the function exceeds a given value. The level set of function $f : \mathbf{R}^d \to \mathbf{R}$ at level $\lambda \in \mathbf{R}$ is defined as

$$\Lambda(f, \lambda) = \left\{ x \in \mathbf{R}^d : f(x) \geq \lambda \right\}. \tag{5.1}$$

Level set tree





Level set tree

Definition 5.2 A level set tree *of function* $\mathbf{R}^d \to \mathbf{R}$, *associated with set of levels* $\mathcal{L} = \{\lambda_1 < \cdots < \lambda_L\}$, *is a tree whose nodes are associated with subsets of* $\mathbf{R}^d$ *and levels in* $\mathcal{L}$ *in the following way.*

1. *Write*

$$\Lambda(f, \lambda_1) = A_1 \cup \cdots \cup A_K,$$

where sets A_i *are pairwise separated, and each is connected. The level set tree has* K *root nodes which are associated with sets* A_i, $i = 1, \ldots, K$, *and each root node is associated with the same level* λ_1.

2. *Let node* m *be associated with set* $B \subset \mathbf{R}^d$ *and level* $\lambda_l \in \mathcal{L}$, $1 \leq l < L$.

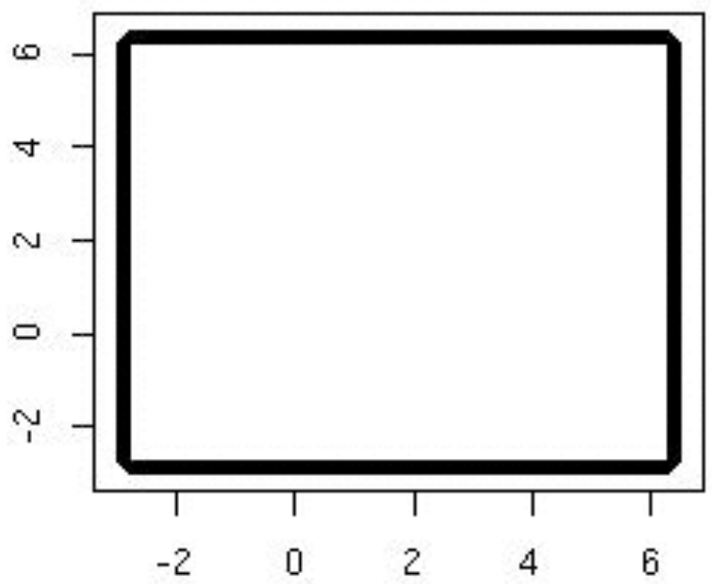
   (a) *If* $B \setminus \Lambda(f, \lambda_{l+1}) = \emptyset$, *then node* m *is a leaf node.*

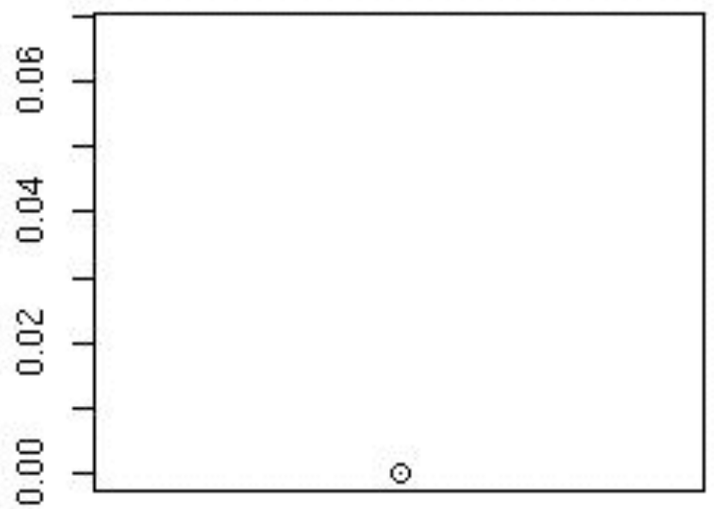
   (b) *Otherwise, write*

$$B \setminus \Lambda(f, \lambda_{l+1}) = C_1 \cup \cdots \cup C_M,$$

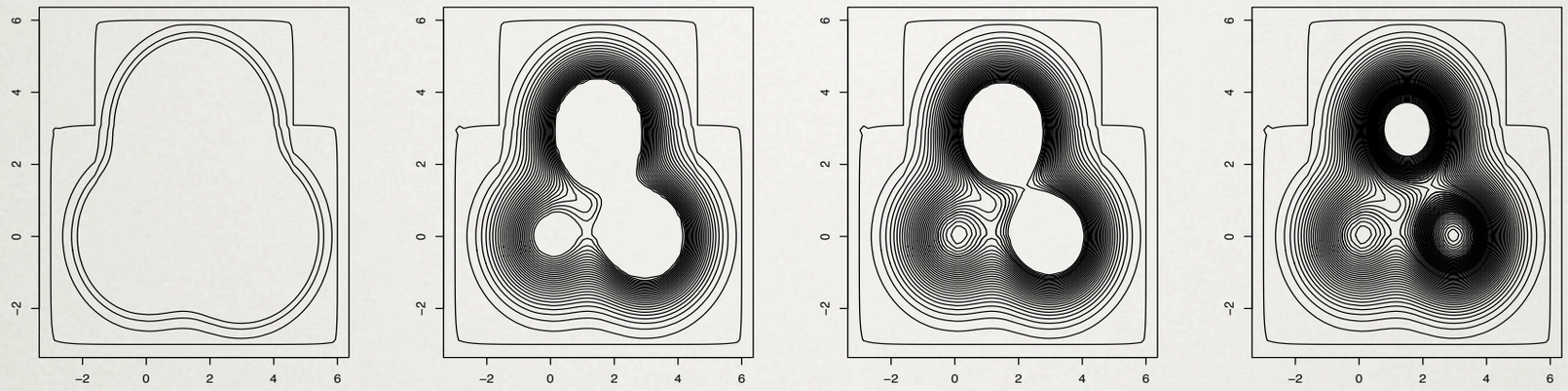
where sets C_i *are pairwise separated, and each is connected. Then node* m *has* M *children, which are associated with sets* C_i, $i = 1, \ldots, M$, *and each child is associated with the same level* λ_{l+1}.

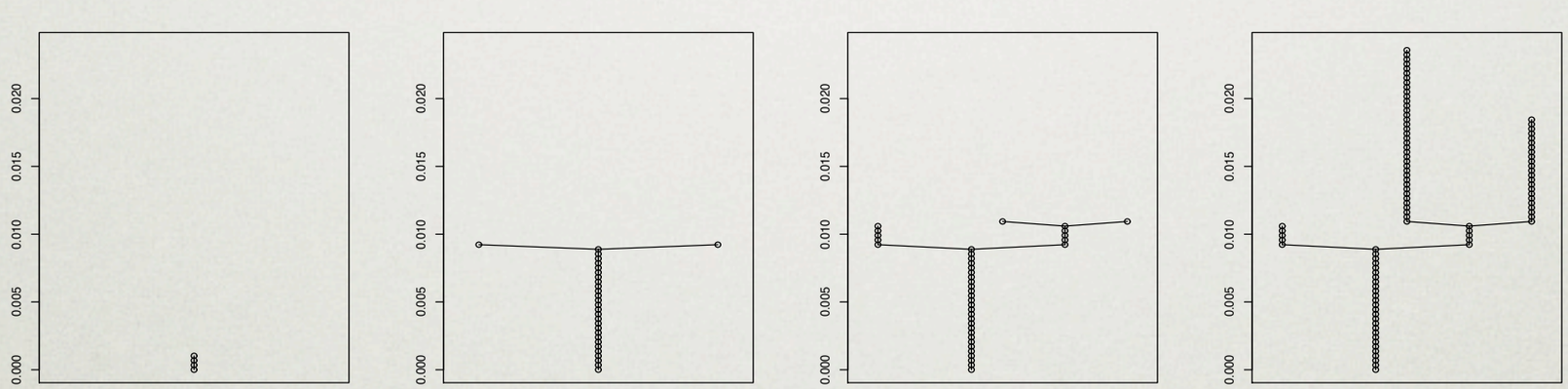
Level set tree





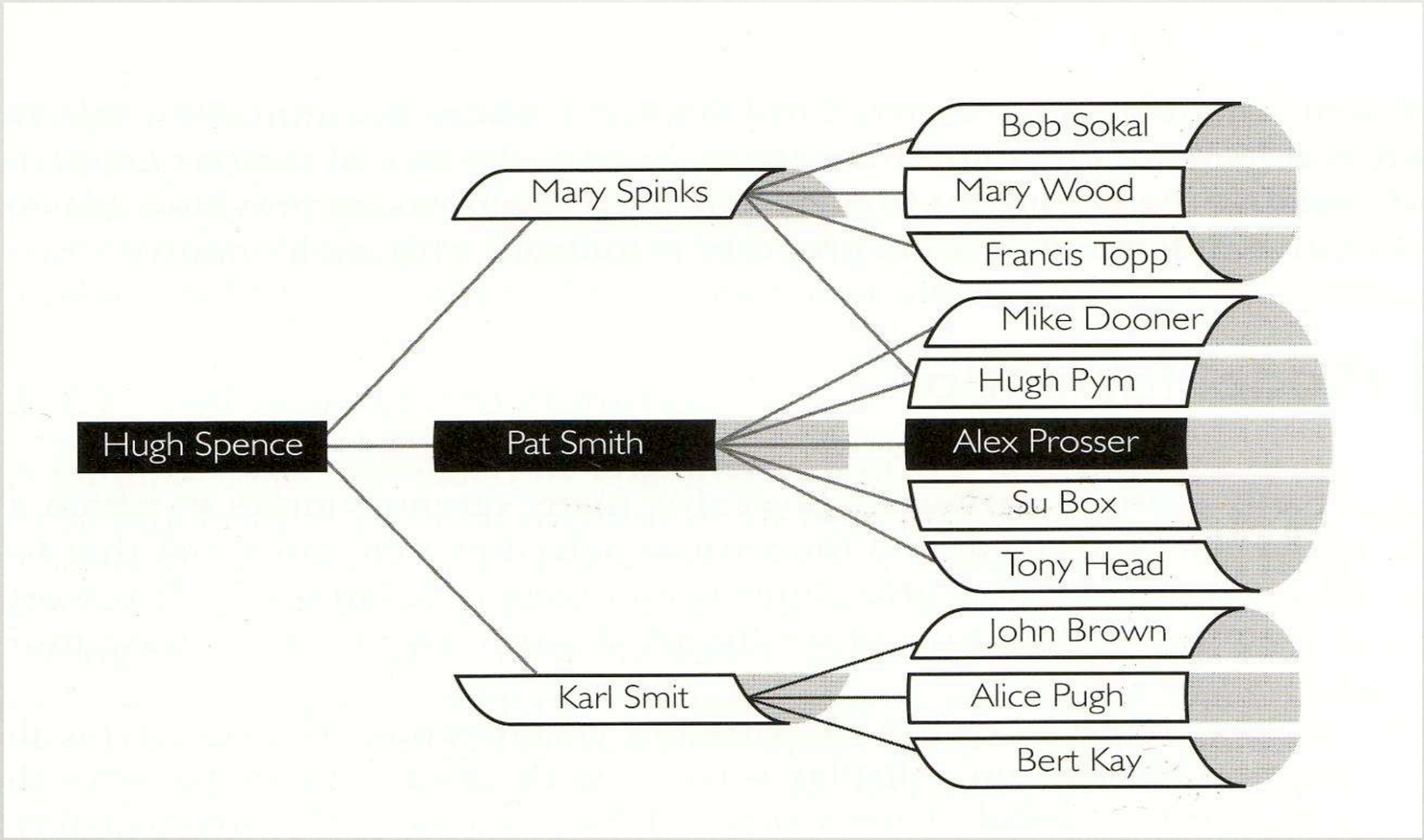
Level set tree



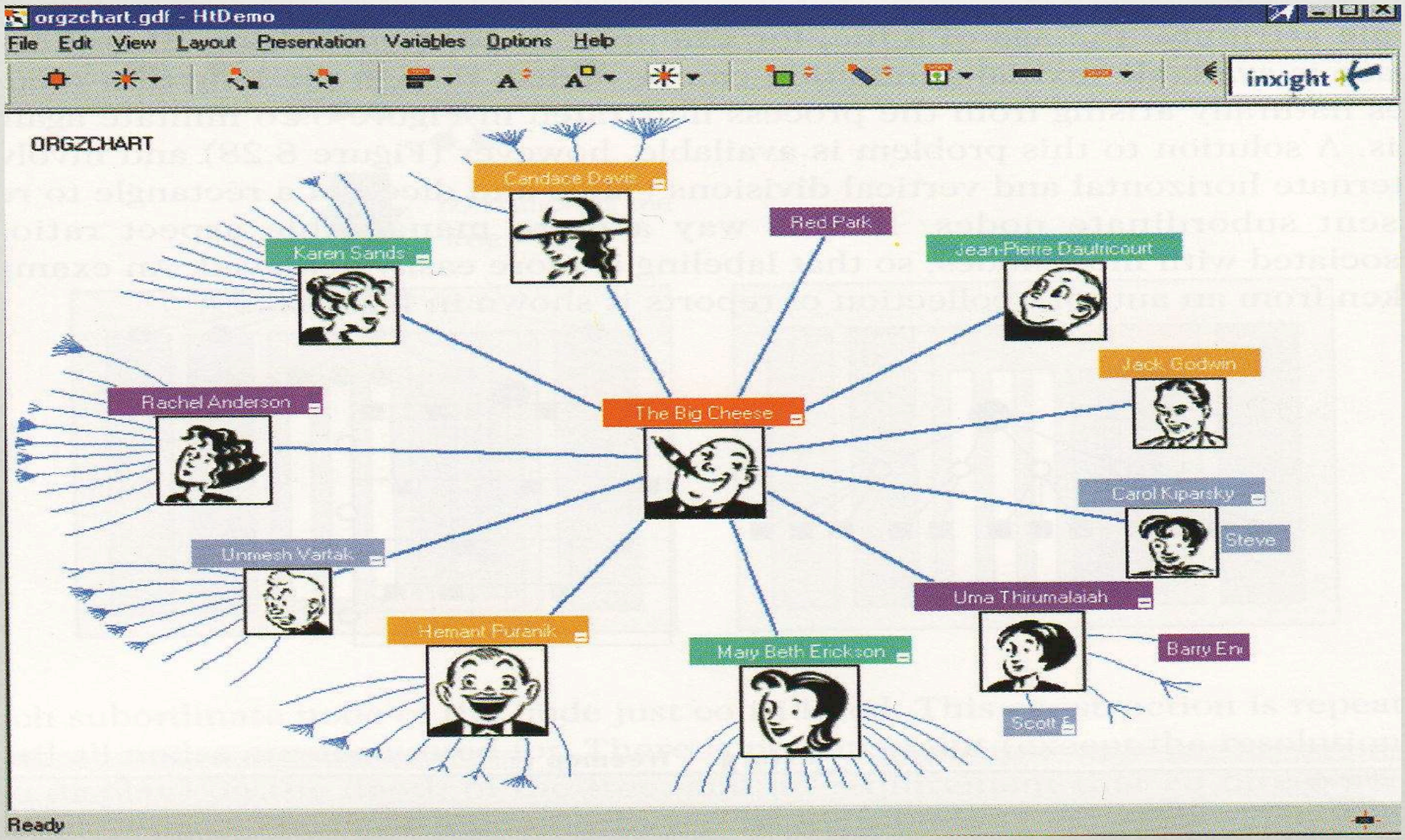


VISUALIZATION OF TREES

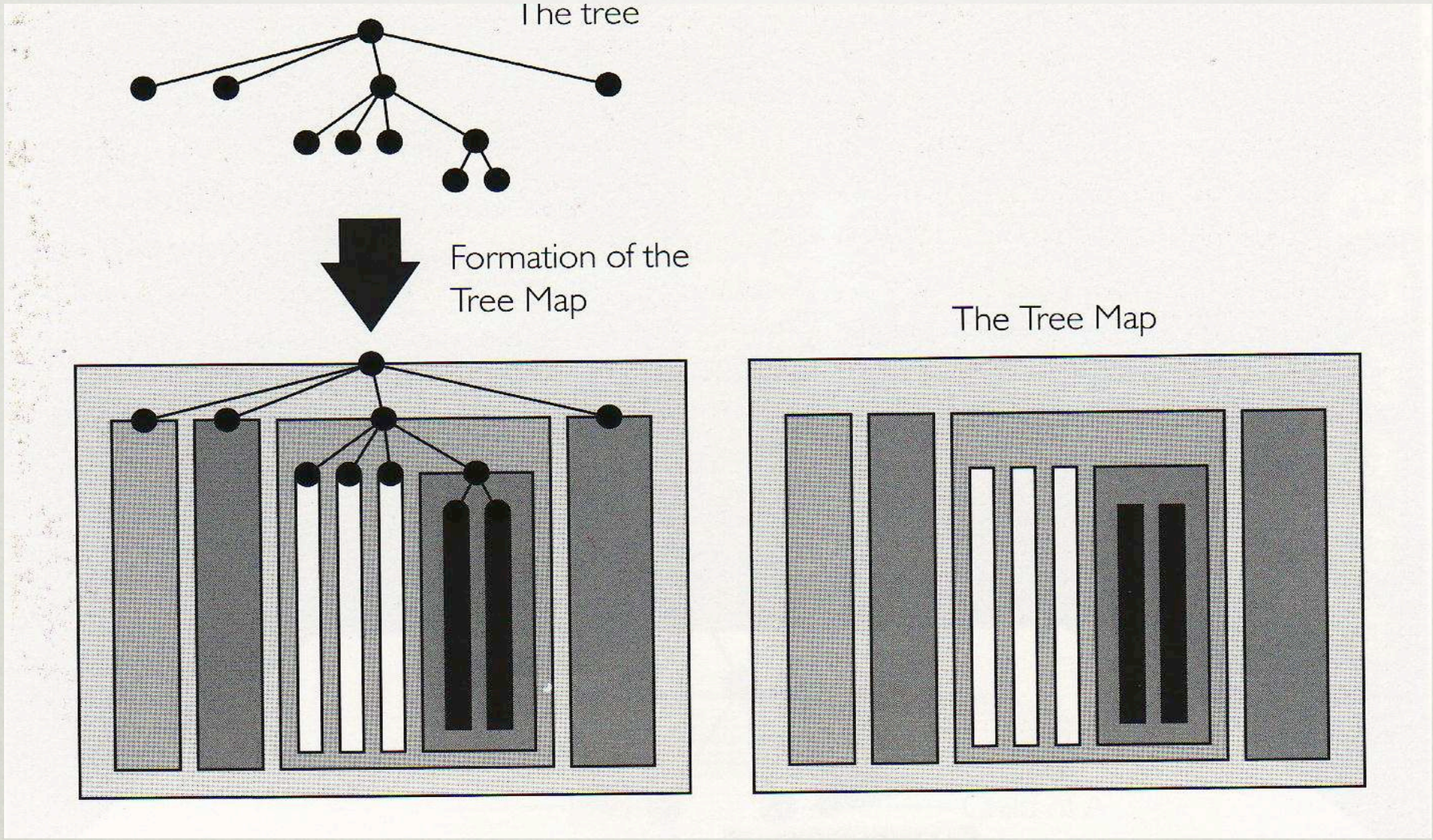
Cone tree, Robert Spence



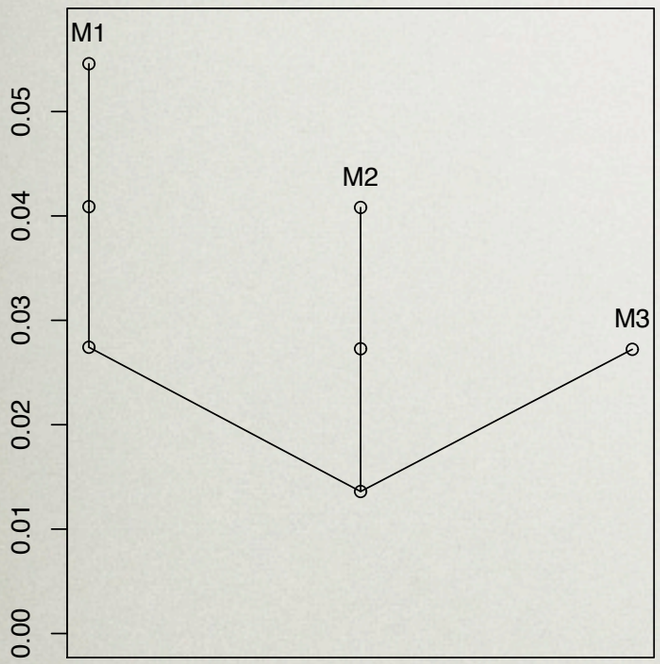
Hyperbolic tree, Inxight Software

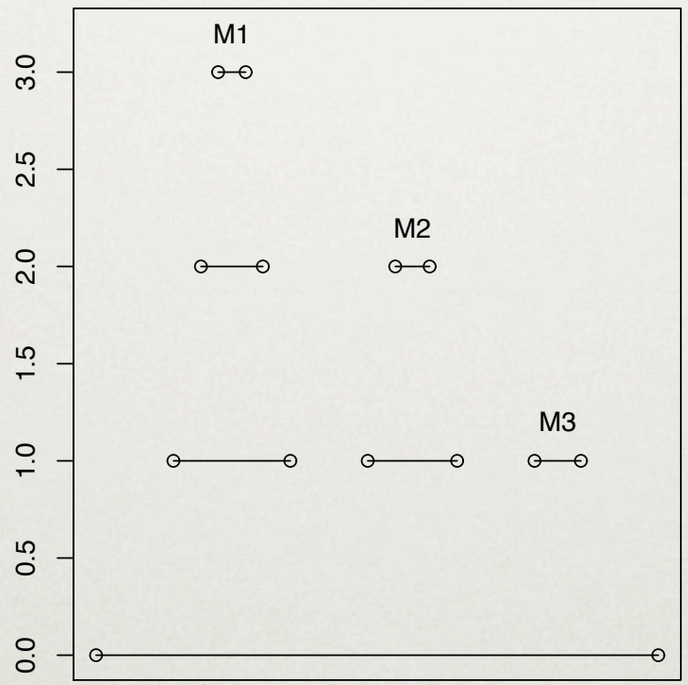


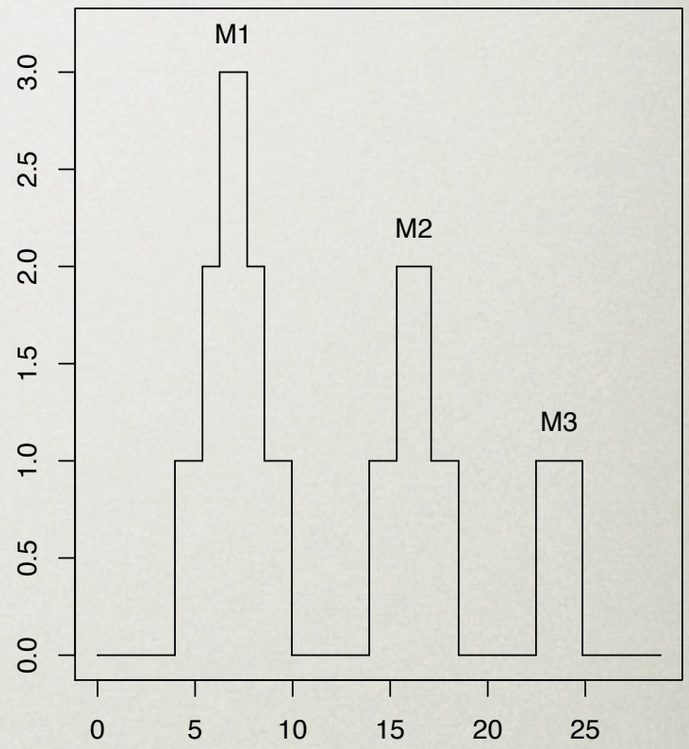
A Tree Map
Ben Shneiderman



Function tree

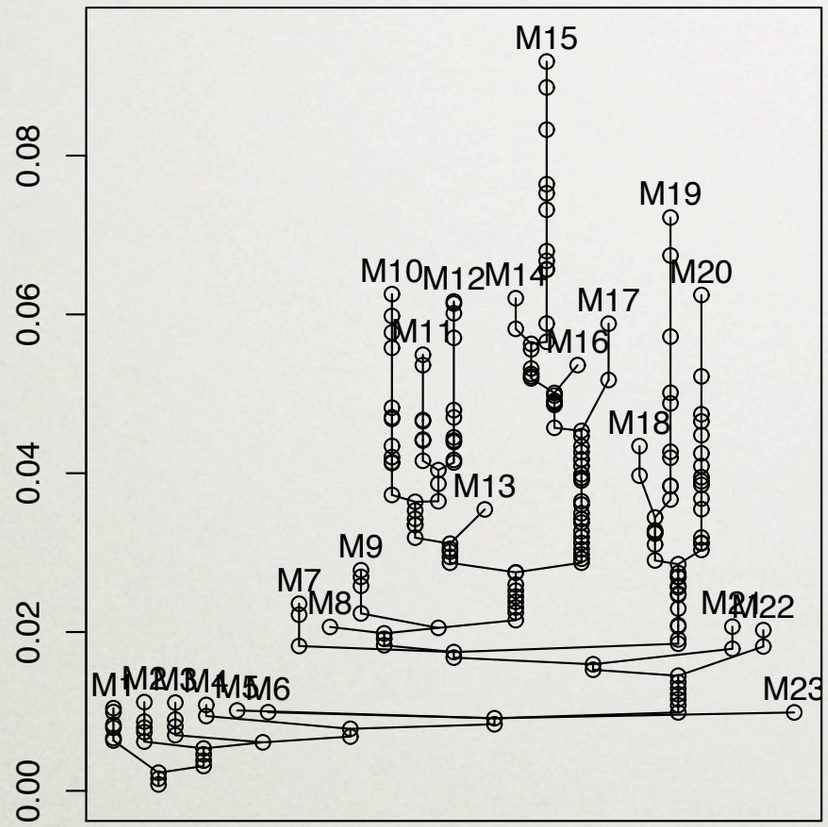


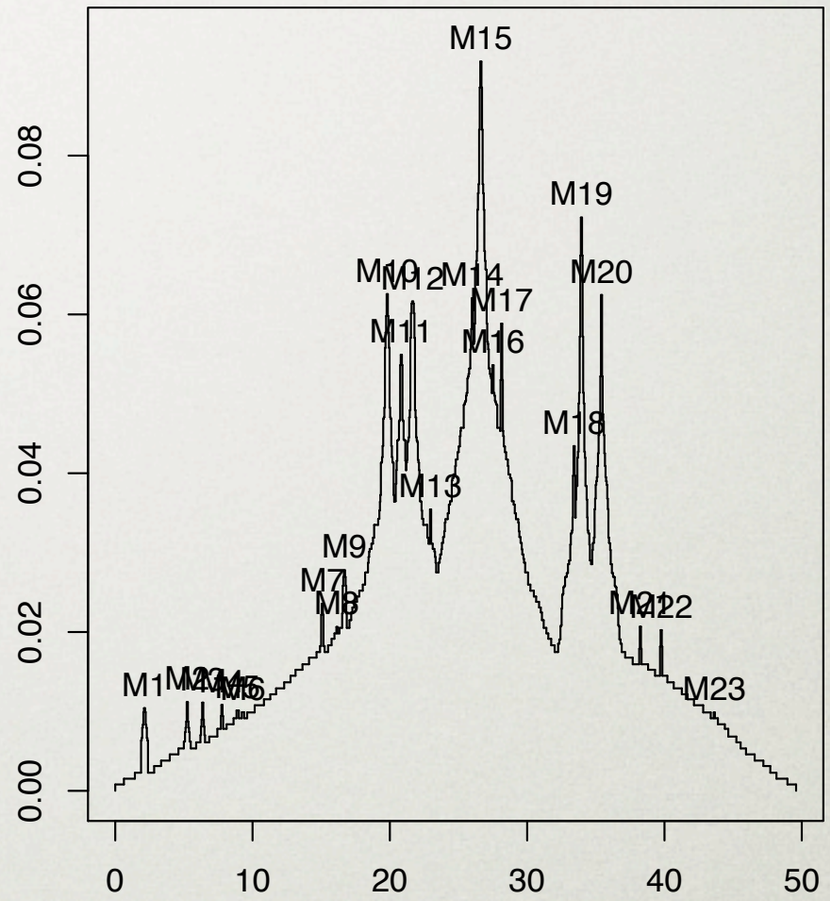




A tree and its function

tree plot





Shape isomorphic visualization

Continued

VOLUME TRANSFORM

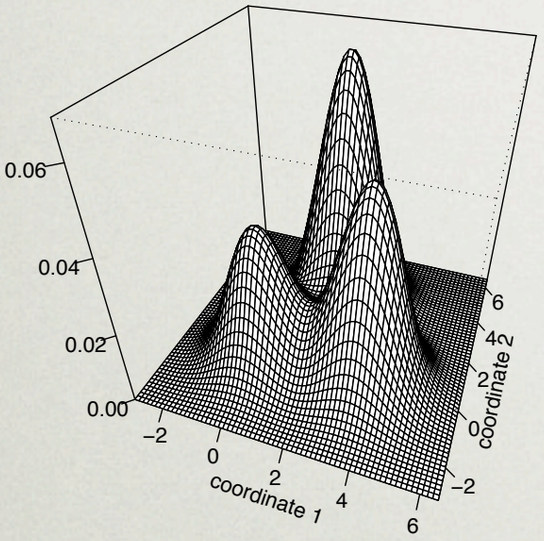


Fig. 1. A perspective plot of a 3-modal function.

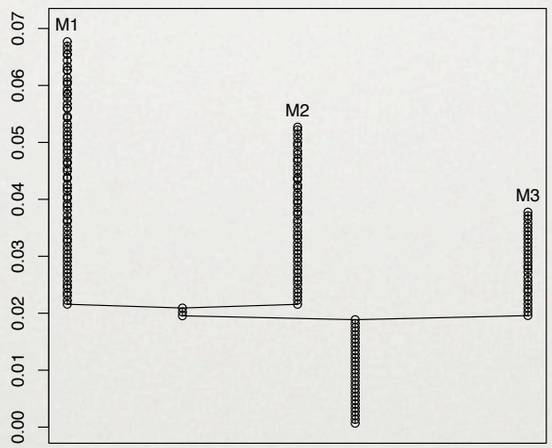


Fig. 2. A level set tree of the 3-modal function.

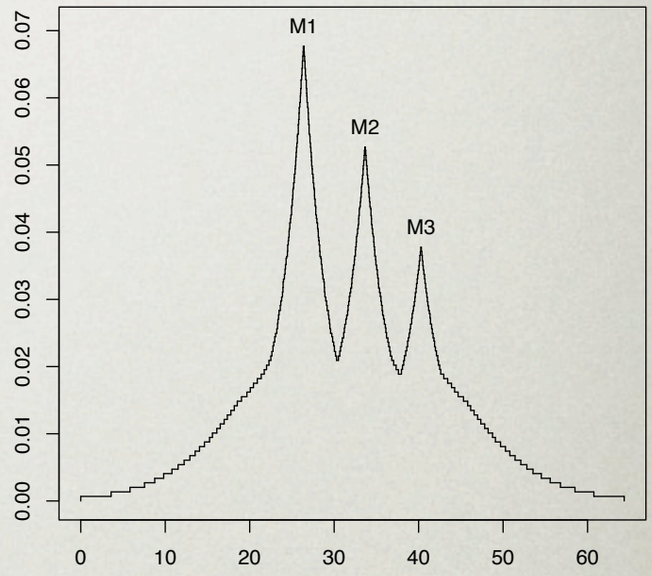
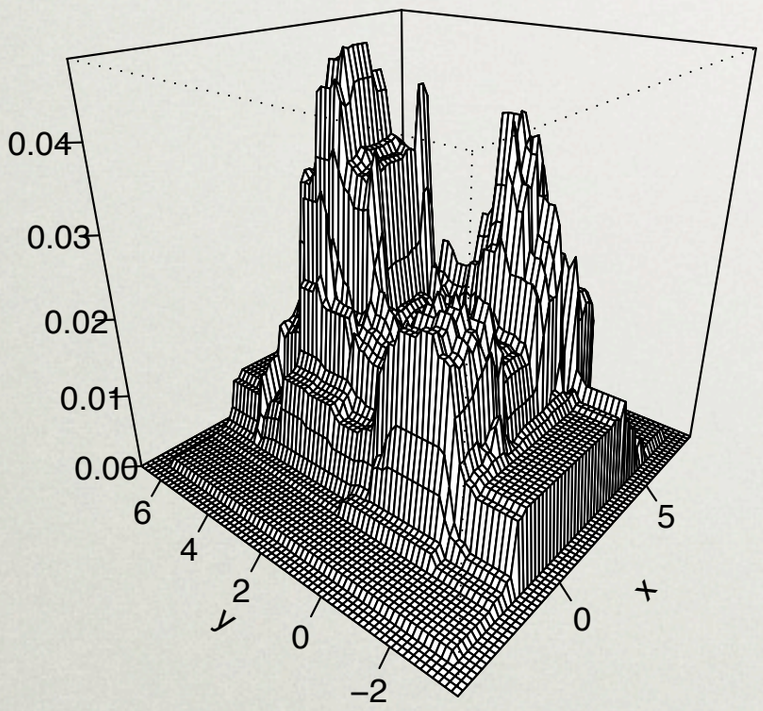
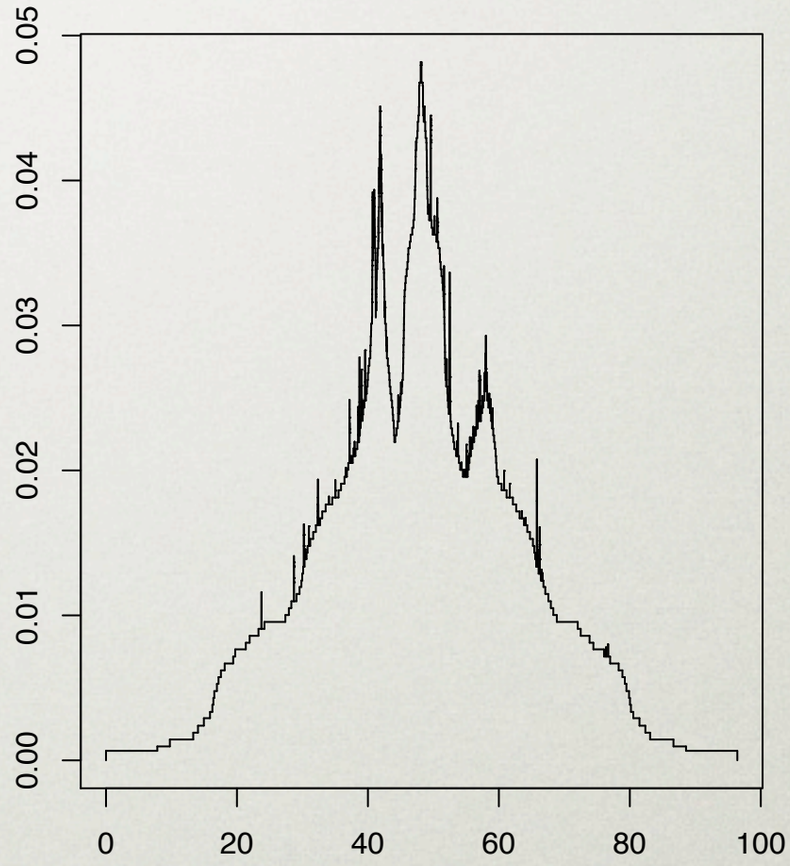


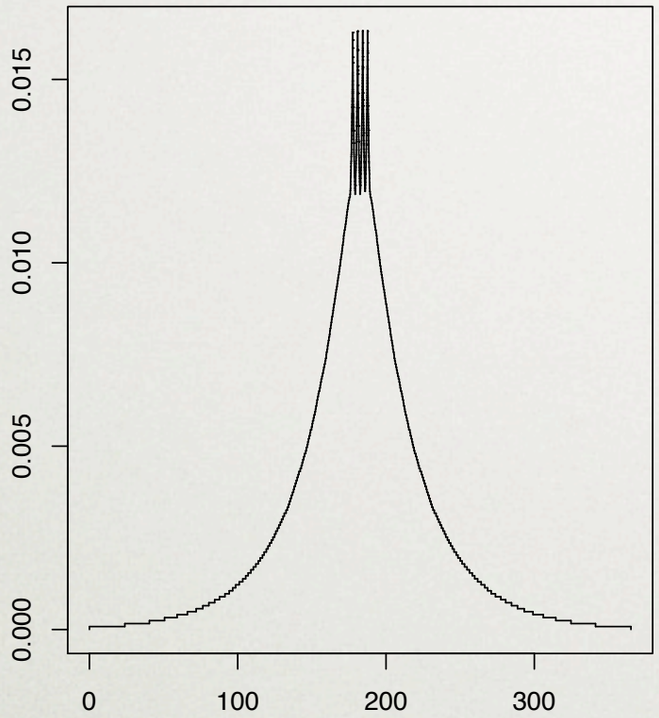
Fig. 3. A volume transform of the 3-modal function.

Example

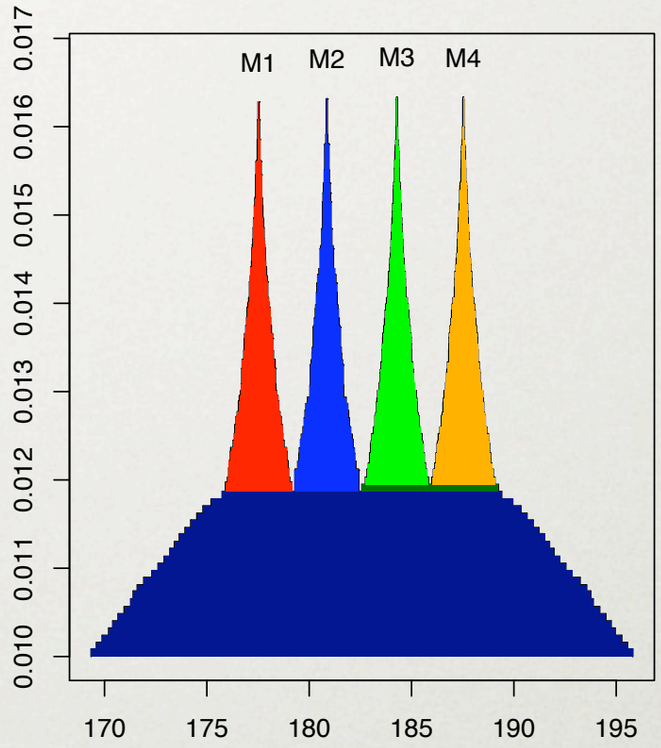

0.04
0.03
0.02
0.01
0.00
6
4
2
0
−2
y
5
0
x


0.05
0.04
0.03
0.02
0.01
0.00
0
20
40
60
80
100

A mixture of 4 3D Gaussians

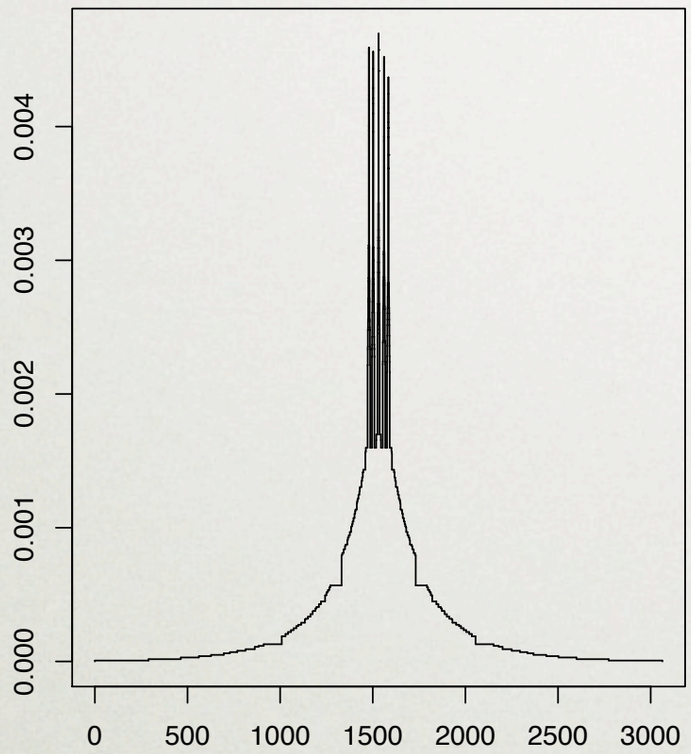


a) volume plot

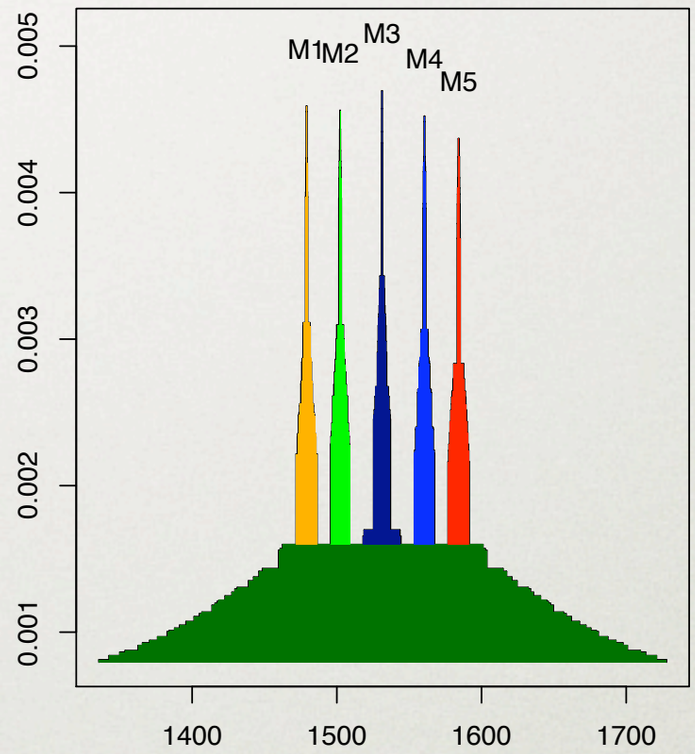


b) zoomed volume plot

A Mixture of 5 4D Gaussians



a) volume plot



b) zoomed volume plot