Portfolio Selection using Kernel Regression

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ABSTRACT

- We use kernel regression to improve the performance of indexes
- Utilizing recent price history can improve the returns

Portfolio Selection

- We have d assets $S_t^1, \ldots, S_t^d$
- Return vector

$$R_t = \left((S_t^1 - S_{t-1}^1)/S_{t-1}^1, \ldots, (S_t^d - S_{t-1}^d)/S_{t-1}^d\right)$$

- Portfolio vector $b_{t-1} \in \mathbf{R}^d$
- Portfolio returns

$$b_{t-1}^T R_t = b_{t-1}^1 R_t^1 + \cdots + b_{t-1}^d R_t^d$$

Markowitz and Expected Utility

- Markowitz: Find portfolio vector b that maximizes

$$E\left(b^T R_{t+1}\right) - \frac{\gamma}{2}\,\mathrm{Var}\left(b^T R_{t+1}\right)$$

- Expected utility: Find b that maximizes

$$Eu\left(b^T R_{t+1}\right)$$

Utility Functions

- CRRA utility functions

$$u(w) = \begin{cases} \frac{w^{1-\gamma}}{1-\gamma}, & \text{if } \gamma > 1, \\ \log_e w, & \text{if } \gamma = 1. \end{cases}$$

- CARA utility functions

$$u(w) = 1 - e^{-\alpha w}, \qquad \alpha > 0.$$

Conditional Expected utility

- Define the conditional expected utility

$$f_b(x) = E\left(u(b^T R_{t+1}) \,|\, X_t = x\right)$$

- Find the portfolio vector as

$$b_t = \mathrm{argmax}_b f_b(x_t)$$

- The explanatory variables can be chosen as

$$X_t = (R_t, \ldots, R_{t-k+1}) \in \mathbf{R}^{kd}$$

Data Based Investment

- We have data for $t = 0, \ldots, t_0 - 1$

$$Y_t = u(b^T R_{t+1}), \;\; X_t = (R_t, \ldots, R_{t-k+1})$$

- We estimate the regression function by

$$\hat{f}_b(x) \approx E\left(u(b^T R_{t+1}) \,|\, X_t = x\right)$$

- Define the portfolio vector as

$$\hat{b}_{t_0} = \mathrm{argmax}_b \hat{f}_b(x_{t_0})$$

Literature

- Ait-Sahalia and Brandt (2001). Variable Selection for Portfolio Choice.
- Györfi, Lugosi, and Udina (2006). Nonparametric Kernel-based Sequential Investment Strategies.
- Andriyashin, Härdle, and Timofeev (2008). Recursive Portfolio Selection with Decision Trees.

Kernel Regression

- Kernel estimate is a weighted average

$$\hat{f}(x) = \sum_{t=0}^{t_0-1} p_t(x)\, Y_t$$

- Weights are defined as

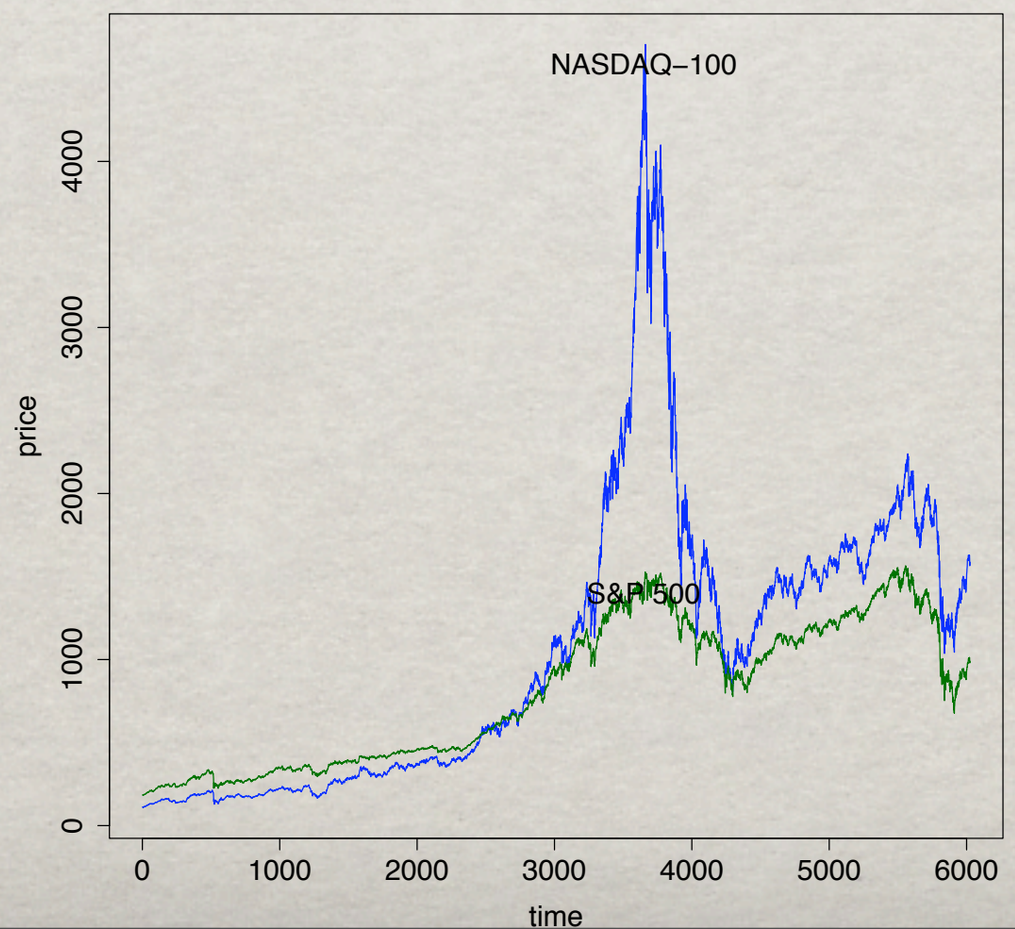
$$p_t(x) = \frac{K_h(x - X_t)}{\sum_{u=0}^{t_0-1} K_h(x - X_u)}$$

- Scaled kernel

$$K : \mathbf{R}^d \to \mathbf{R}, \;\; K_h(x) = K(x/h)/h^d$$

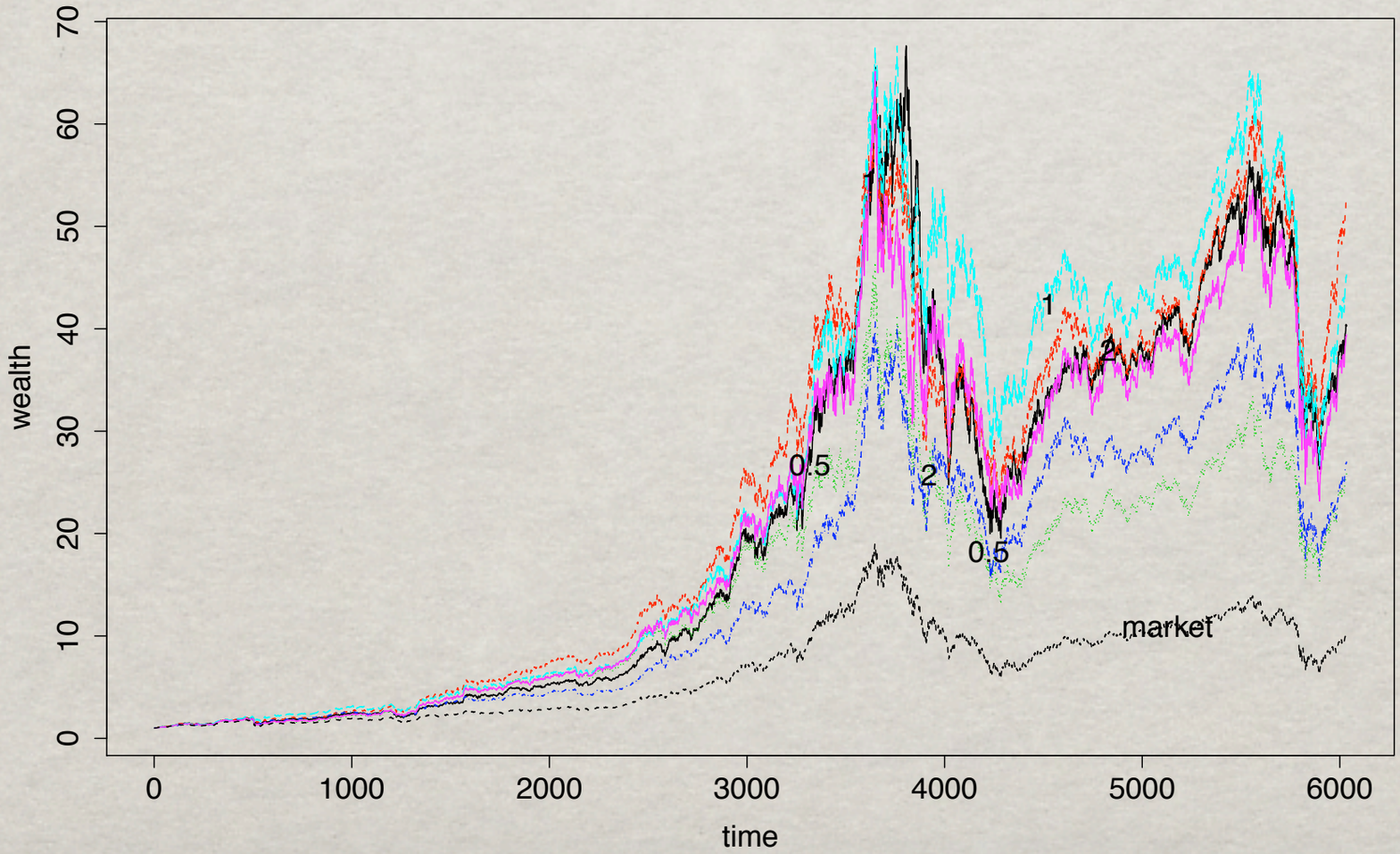
S&P 500, Nasdaq-100

- Data between 1985-01-01 and 2009-05-04
- S&P 500: annualized mean 8.8%, standard deviation 18.9%, Sharpe ratio 0.47
- NASDAQ-100: annualized mean 15.2%, standard deviation 28.8%, Sharpe ratio 0.53



S&P 500, Nasdaq-100

- Smoothing parameter h = 0.5,1,2
  autoregression parameter k = 5,10.
- Best Sharpe ratio 0.79 and the worst 0.64.



Parameter Selection

- Let us have L parameter combinations that give L sequences of portfolio weights

$$b_0^{(l)}, \ldots, b_{t_0}^{(l)}, \;\; l = 1, \ldots, L$$

- The corresponding wealth processes

$$W_{t_0}^{(l)} = W_0 \prod_{t=0}^{t_0-1} (R_{t+1} + 1)^T b_t^{(l)}$$

- The portfolio vector obtained by combining strategies is

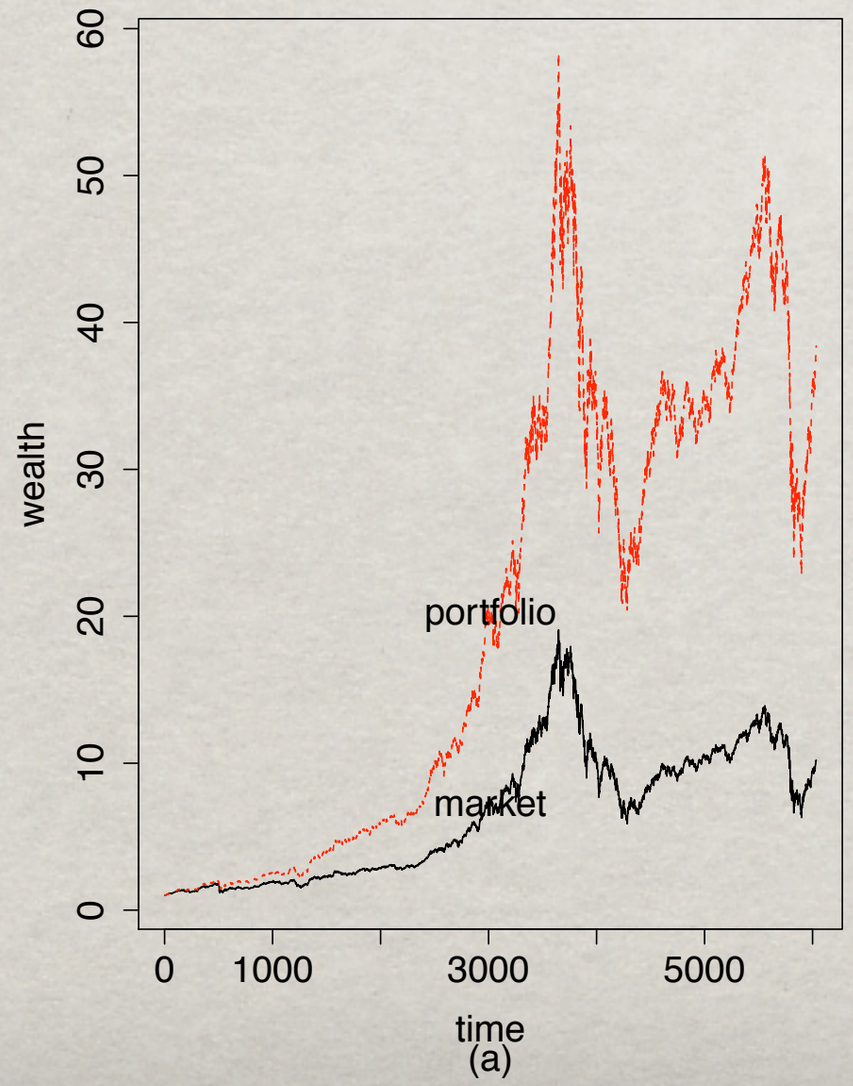
$$b_t = \sum_{l=1}^{L} q_t^{(l)} b_t^{(l)}, \;\; t = 1, \ldots, t_0$$

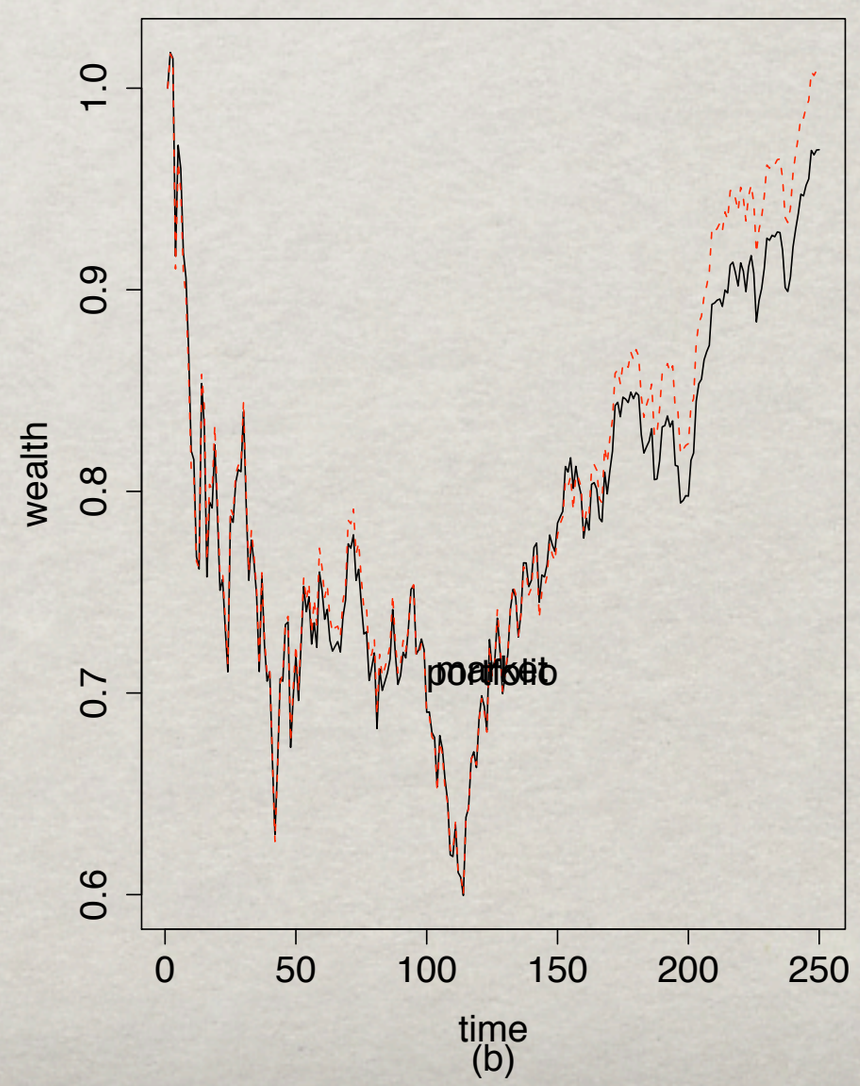
- where

$$q_t^{(l)} = \frac{W_{t-1}^{(l)}}{\sum_{l=1}^{L} W_{t-1}^{(l)}}$$

S&P 500, Nasdaq-100

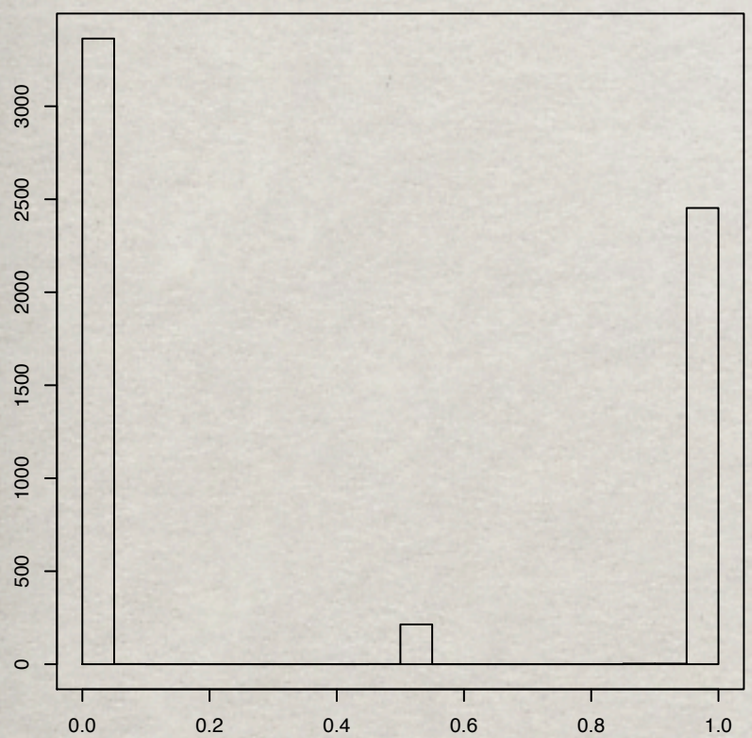
- Long only, without bank account
- Annualized mean is 18.1%, standard deviation 24.3%, Sharpe ratio 0.74.

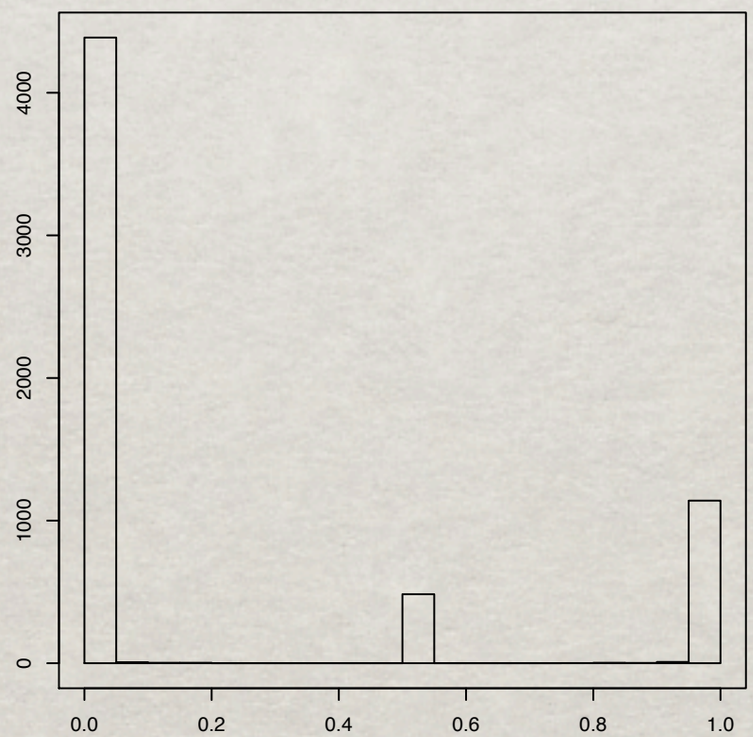


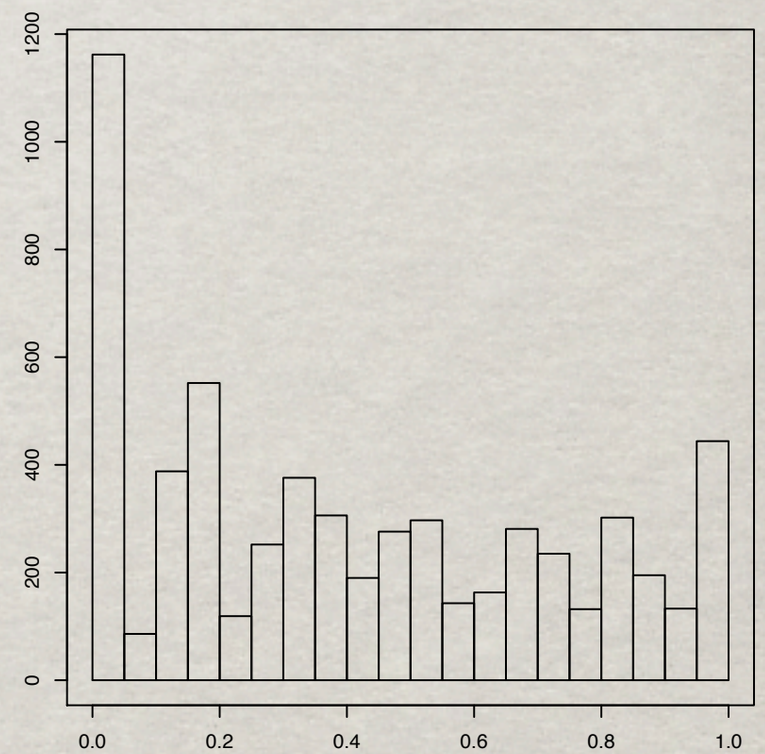


Portfolio Weights

- Simple strategies have mostly 0-1-weights

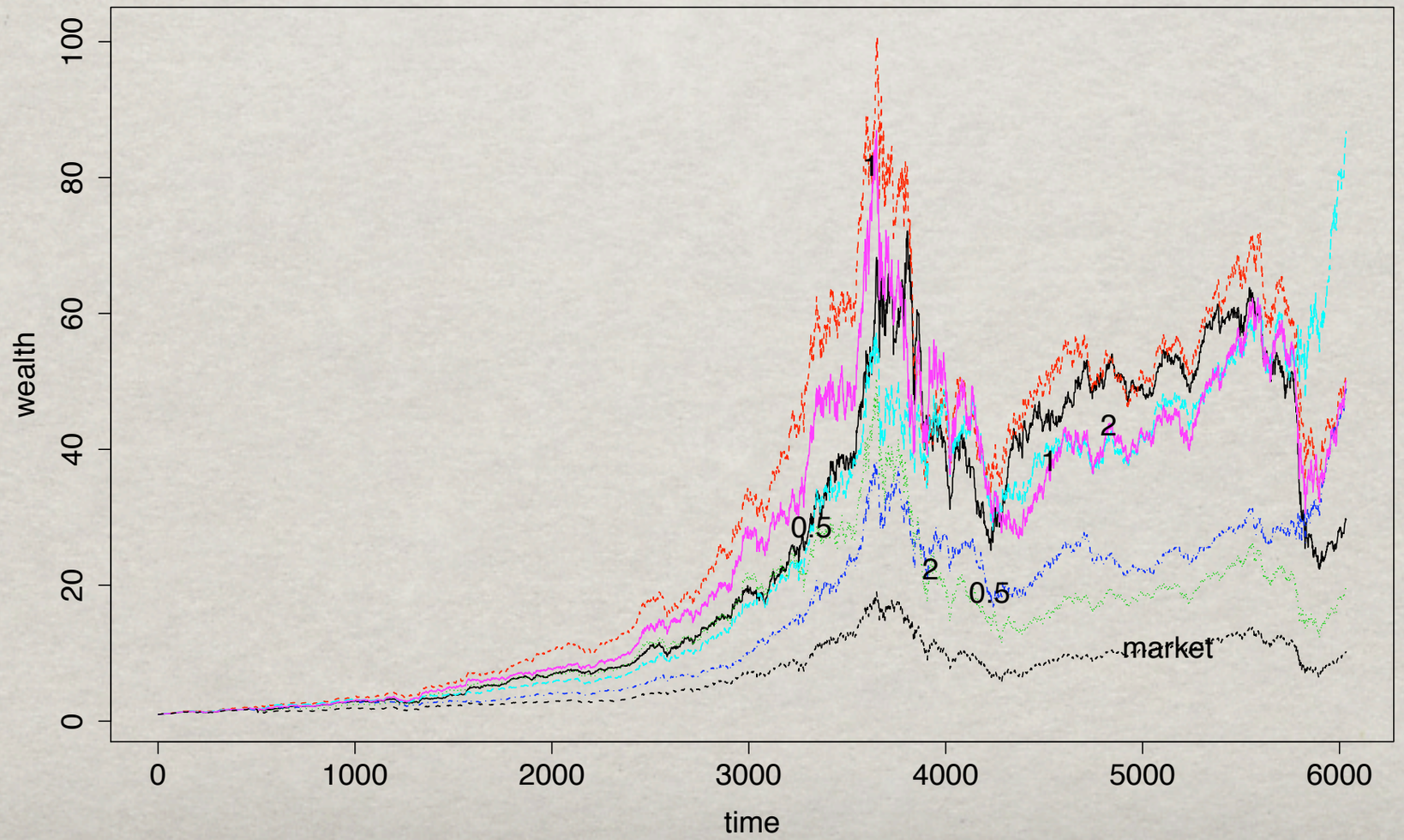






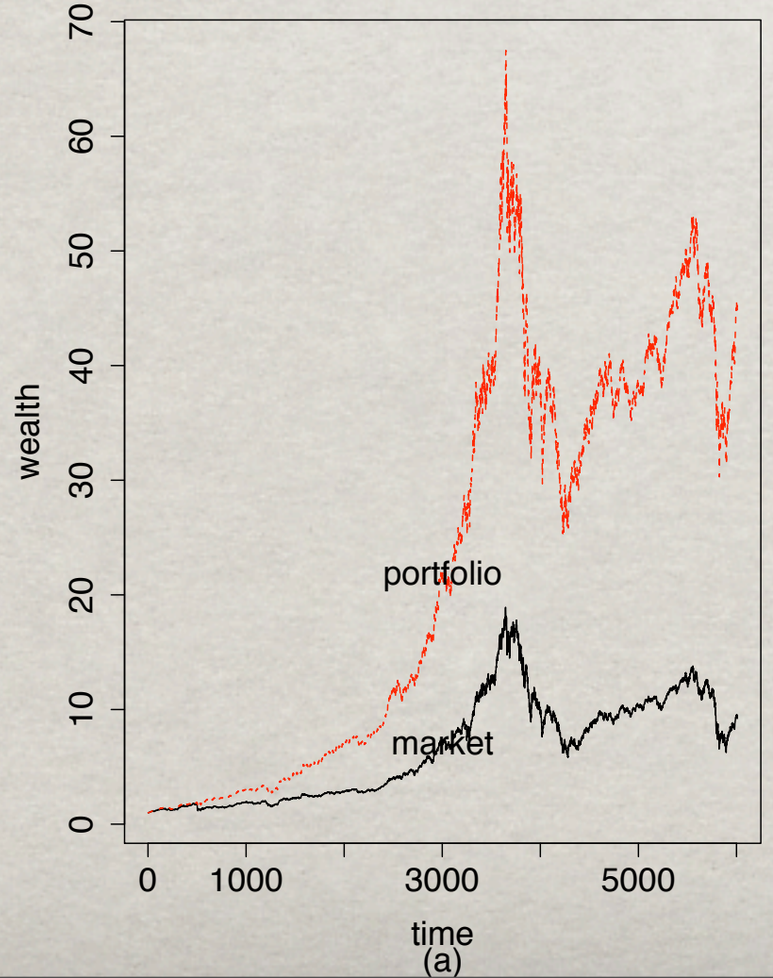
S&P 500, Nasdaq-100

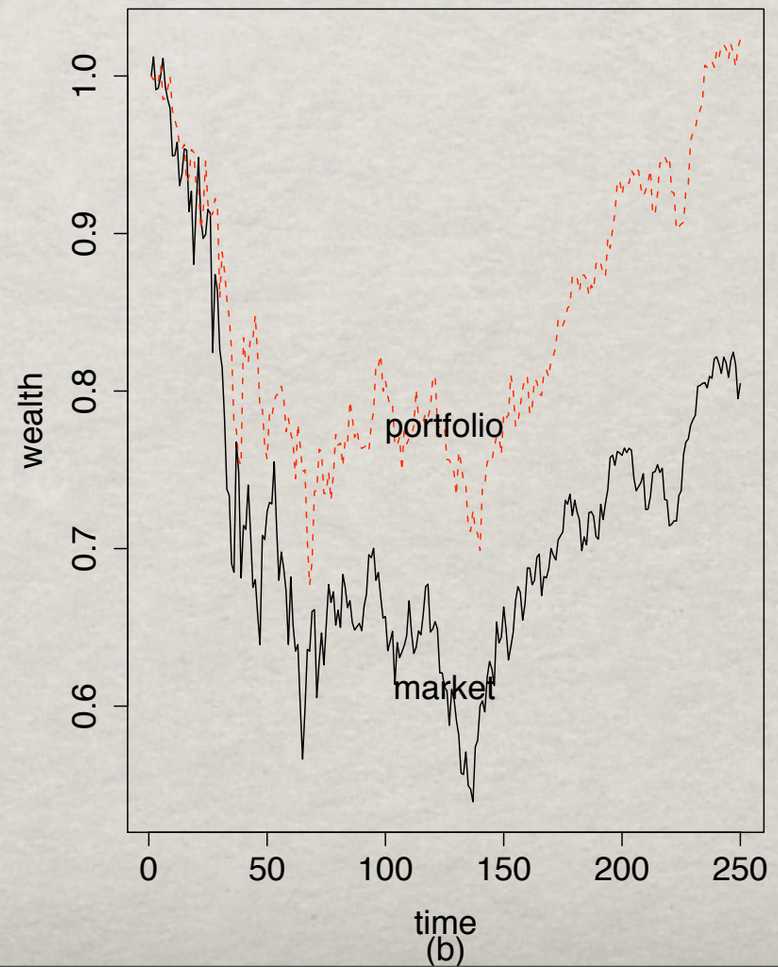
- Bank account included
- Smoothing parameter h = 0.5,1,2
  autoregression parameter k = 5,10.
- Best Sharpe ratio 0.98 and the worst 0.62.



S&P 500, Nasdaq-100

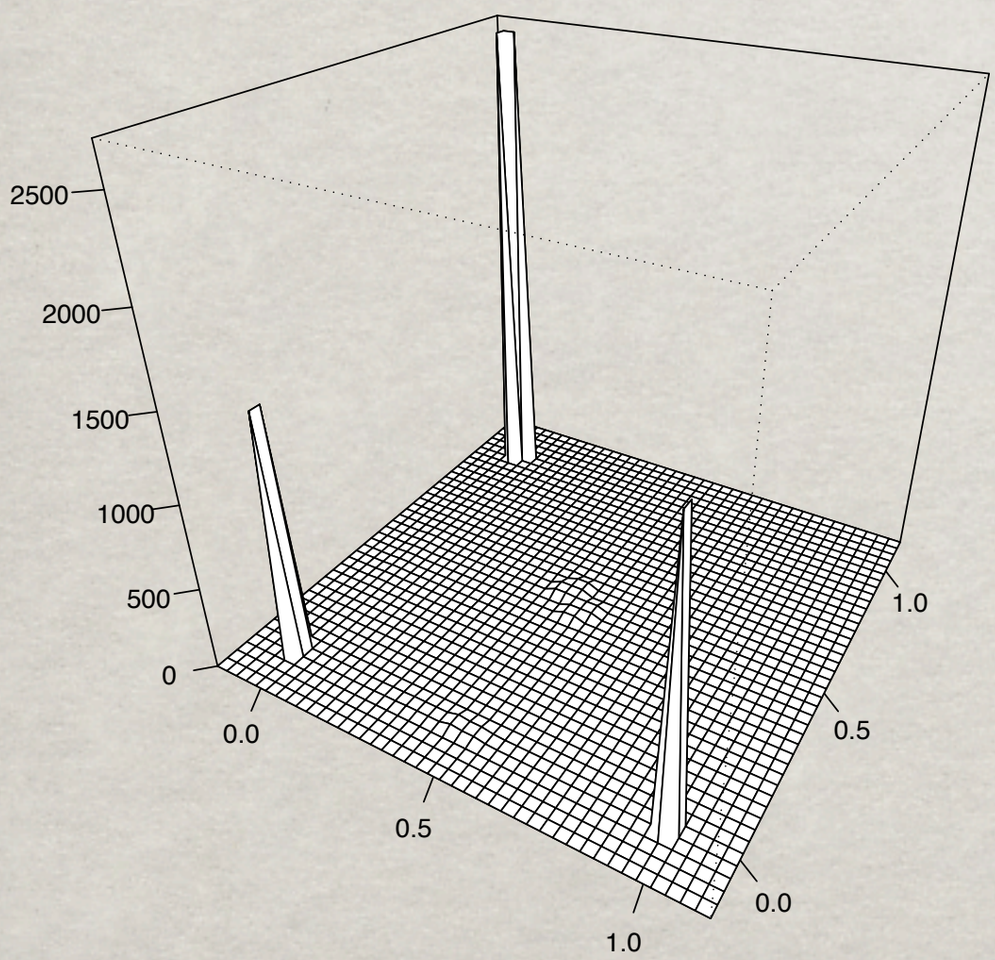
- Combining strategies
- Long only, with bank account
- Annualized mean is 18.1%, standard deviation 20.3%, Sharpe ratio 0.89

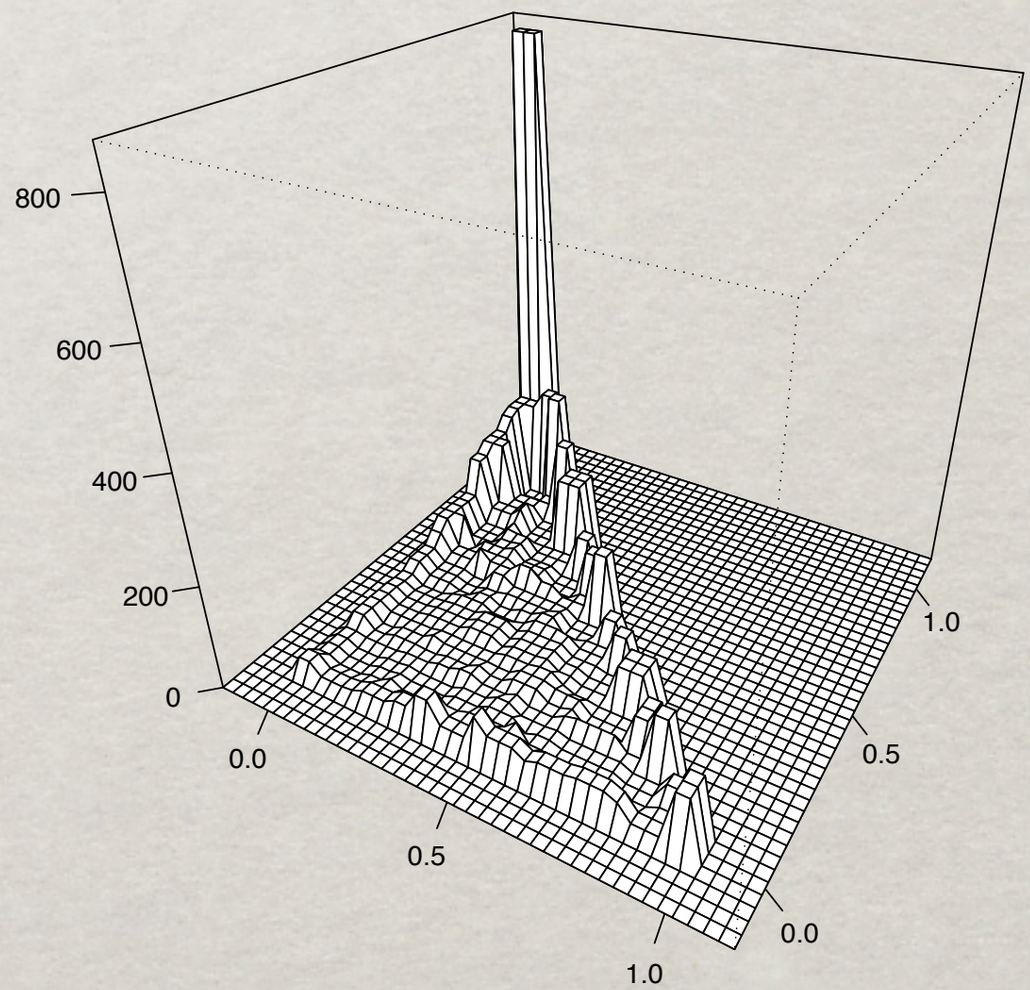




Portfolio Weights

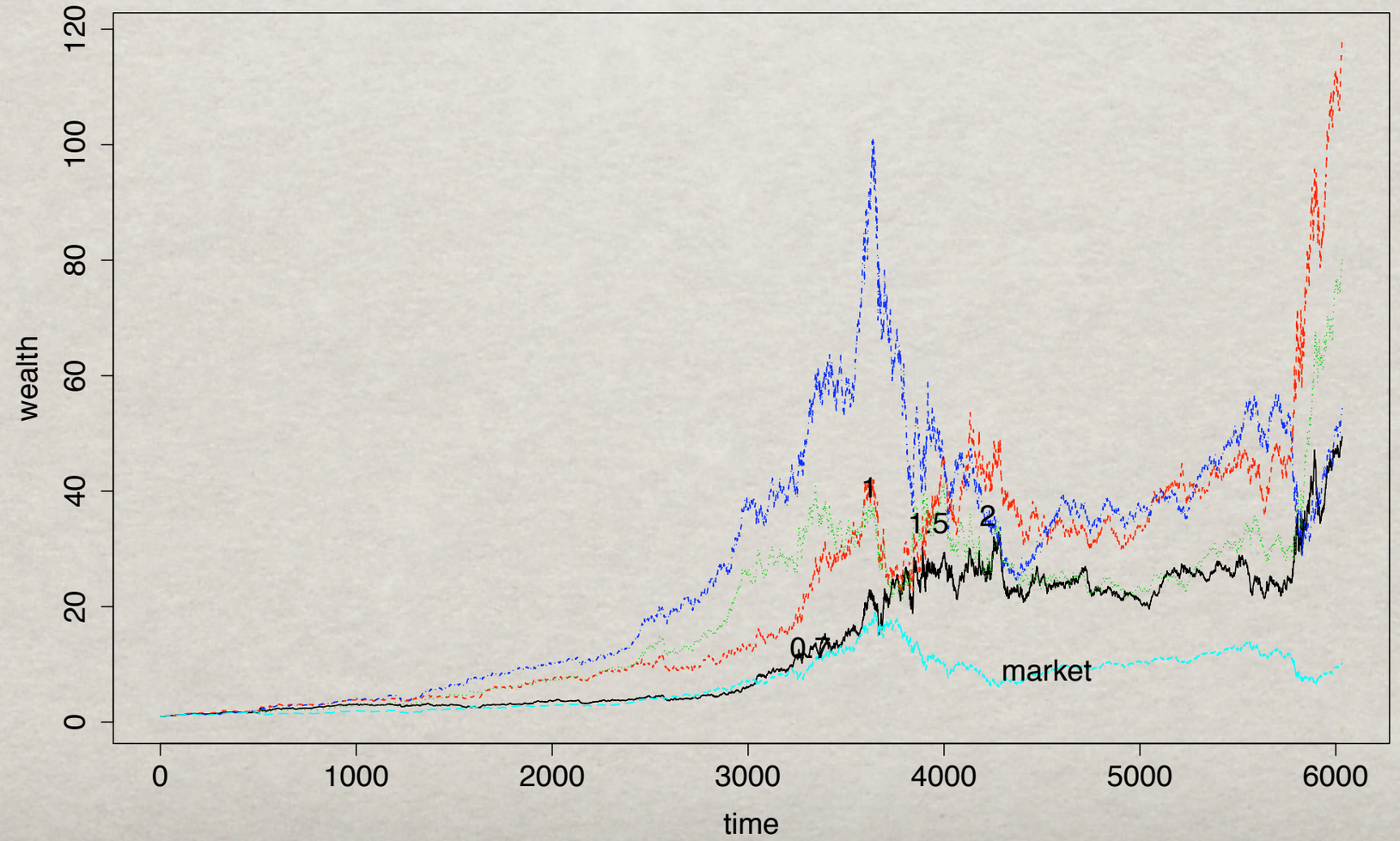
- Simple strategies have mostly 0-1-weights





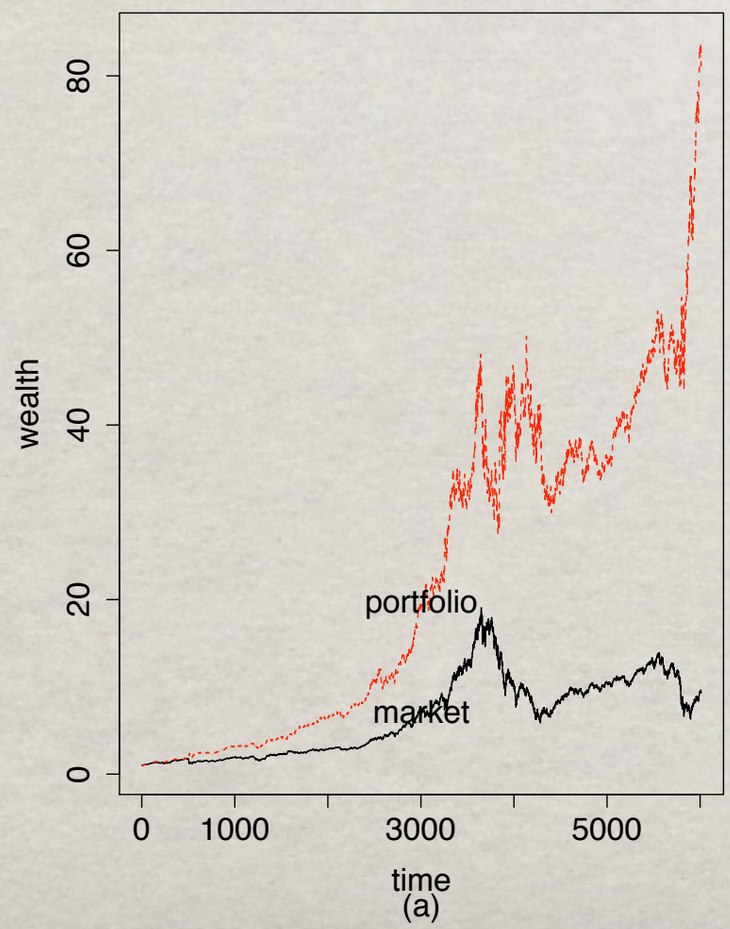
SP 500, Nasdaq-100

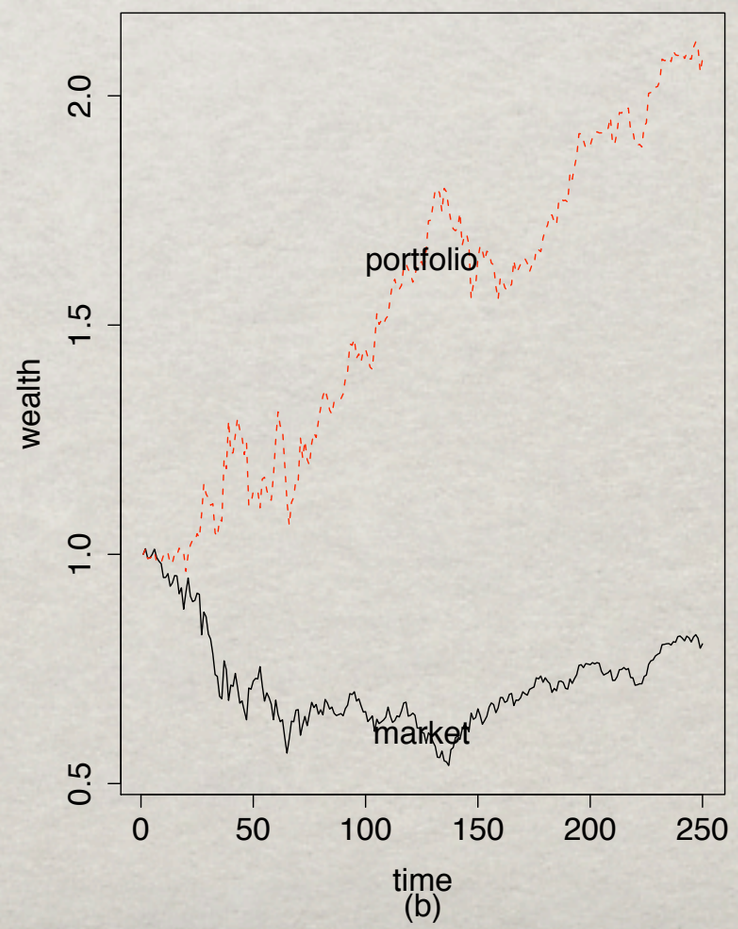
- Shorting allowed
- Smoothing parameter h = 0.7,1,1.5,2
  autoregression parameter k = 10.
- Best Sharpe ratio 0.87 and the worst 0.74.



SP500, Nasdaq-100

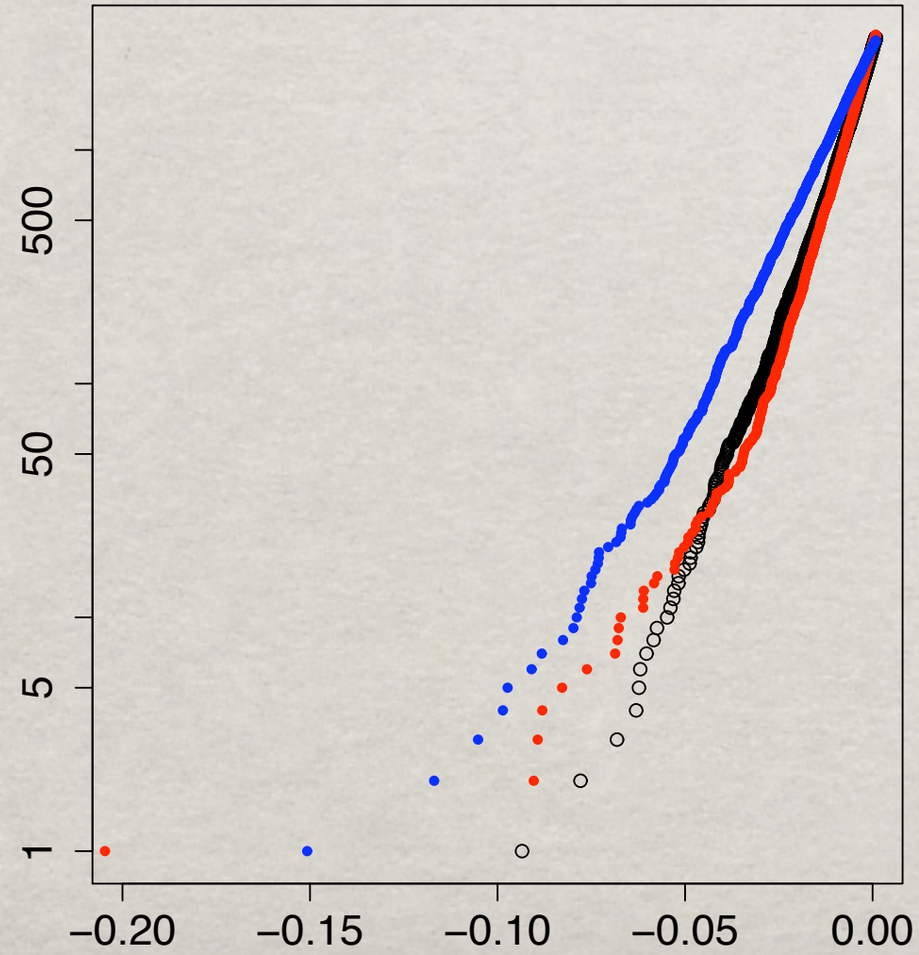
- Combining strategies
- Shorting allowed
- Annualized mean is 20.4%, standard deviation 22.3%, Sharpe ratio 0.91.

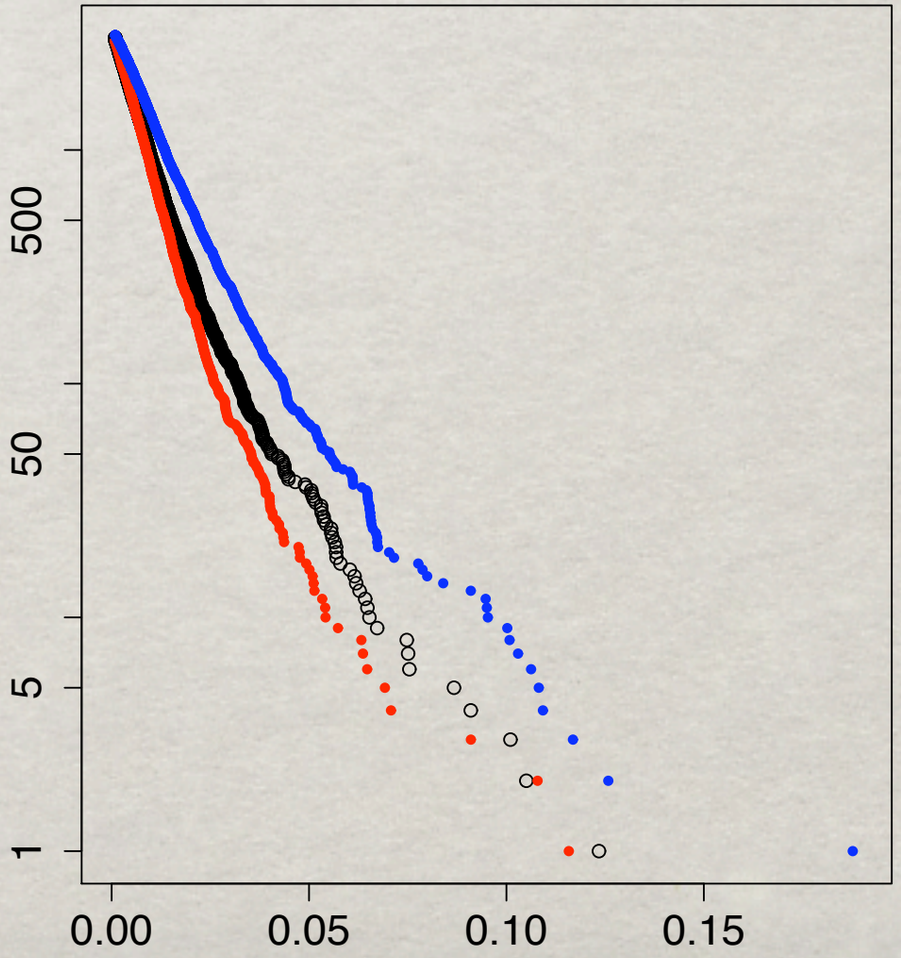




S&P 500, Nasdaq-100

- Combining strategies, shorting allowed
- Left and right tail plots
- red=S&P 500, blue=NASDAQ-100, black=portfolio





Quantiles

0.01	0.05	0.1	0.5	0.9	0.05	0.99
-3.8%	-2.0%	-1.3%	0.06%	1.4%	2.1%	4.4%

Table 6: Quantiles of the SP500–NASDAQ-100 portfolio allowing shorting.

0.01	0.05	0.1	0.5	0.9	0.05	0.99
-2.6%	-1.4%	-1.0%	0.04%	1.0%	1.4%	2.6%

Table 15: Empirical quantiles of the S&P 500 returns.

0.01	0.05	0.1	0.5	0.9	0.05	0.99
-4.9%	-2.8%	-1.9%	0.13%	1.9%	2.8%	5.2%

Table 18: Empirical quantiles of the NASDAQ-100 returns.

Biggest losses

-10.9%	-9.2%	-8.3%	-7.7%	-7.0%
2008-10-28	2000-04-14	2001-09-17	2001-03-28	2000-04-12
-6.8%	-6.7%	-6.7%	-6.5%	-6.5%
2009-03-23	2008-11-20	2001-03-20	2002-07-29	2000-12-08

Table 4: The 10 biggest daily losses of the SP500–NASDAQ-100 portfolio allowing shorting and their dates.

-20.5%	-9.0%	-8.9%	-8.8%	-8.3%
1987-10-19	2008-10-15	2008-12-01	2008-09-29	1987-10-26
-7.6%	-6.9%	-6.8%	-6.8%	-6.7%
2008-10-09	1997-10-27	1998-08-31	1988-01-08	2008-11-20

Table 13: The 10 biggest daily losses of S&P 500 and their dates.

-15.1	-11.7	-10.5	-9.9	-9.7
1987-10-19	1987-10-26	2008-09-29	1998-08-31	2000-04-14
-9.1	-8.8	-8.3	-8.0	-7.9
2001-01-02	2008-10-15	2001-09-17	2008-12-01	2000-12-20

Table 16: The 10 biggest daily losses of NASDAQ-100 and their dates.

Biggest Daily Gains

20.5%	18.8%	12.6%	10.6%	9.8%
1987-10-19	2001-01-03	2008-10-13	2002-05-08	1987-10-21
9.3%	9.10%	9.09%	8.8%	8.4%
1987-10-26	2000-10-13	2001-01-02	2008-10-15	2001-04-05

Table 5: The 10 biggest daily gains of the SP500–NASDAQ-100 portfolio allowing shorting and their dates.

11.6	10.8	9.1	7.1	6.9
2008-10-13	2008-10-28	1987-10-21	2009-03-23	2008-11-13
6.5	6.4	6.3	5.7	5.4
2008-11-24	2009-03-10	2008-11-21	2002-07-24	2008-09-30

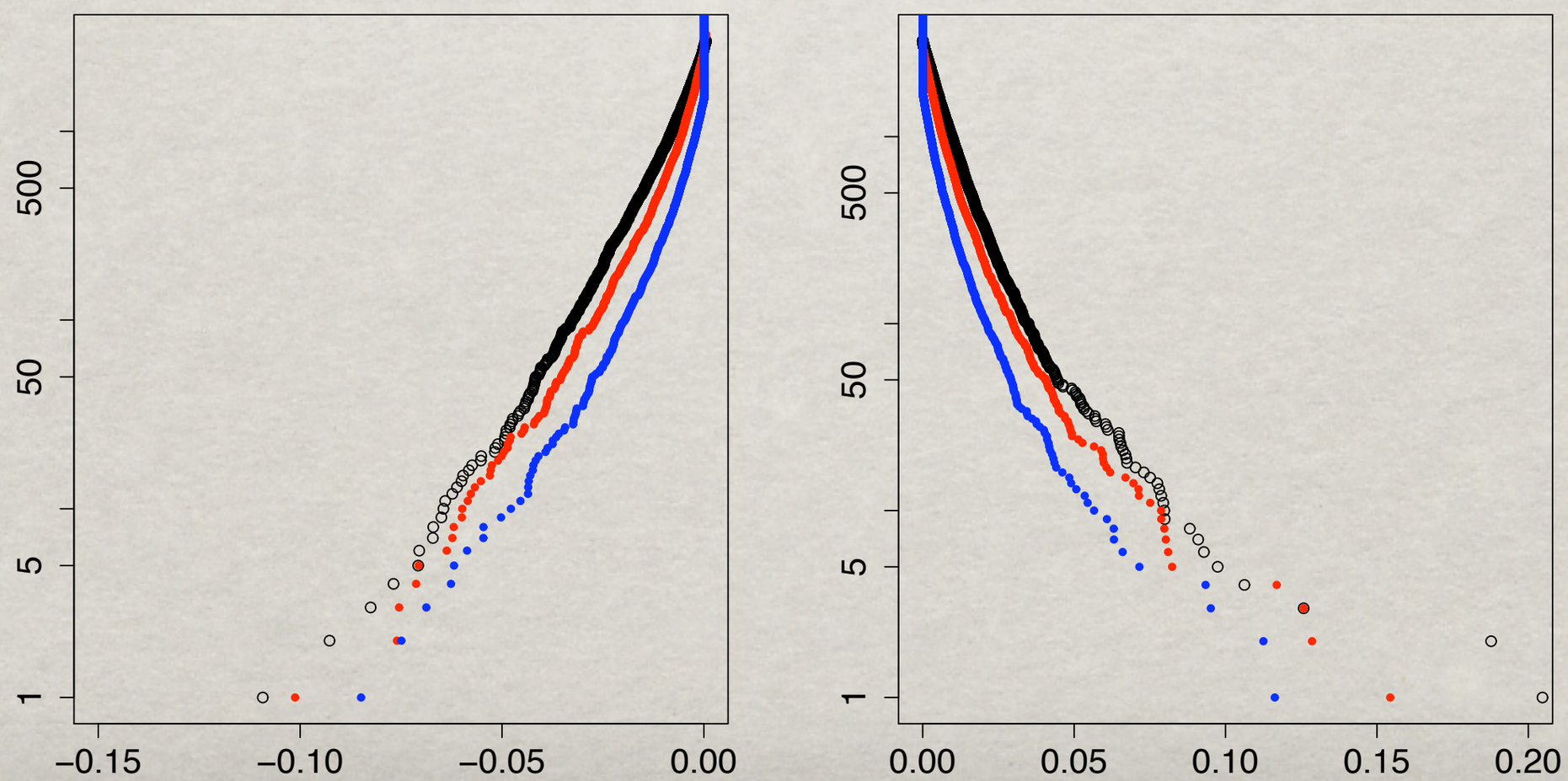
Table 14: The 10 biggest daily gains of S&P 500 and their dates.

18.8	12.6	11.7	10.9	10.8
2001-01-03	2008-10-13	2000-12-05	2008-10-28	2001-04-05
10.6	10.3	10.1	.10.0	9.5
2002-05-08	1987-10-21	2000-05-30	2000-04-17	2001-04-18

Table 17: The 10 biggest daily gains of NASDAQ-100 and their dates.

S&P 500, Nasdaq-100

- Combining strategies, shorting allowed
- Left and right tail plots for different risk aversion parameters gamma
- black: $\gamma = 1$, red: $\gamma = 25$, blue: $\gamma = 100$
- Sharpe ratios: 0.91, 0.87, and 0.61



Expected Utility Curves

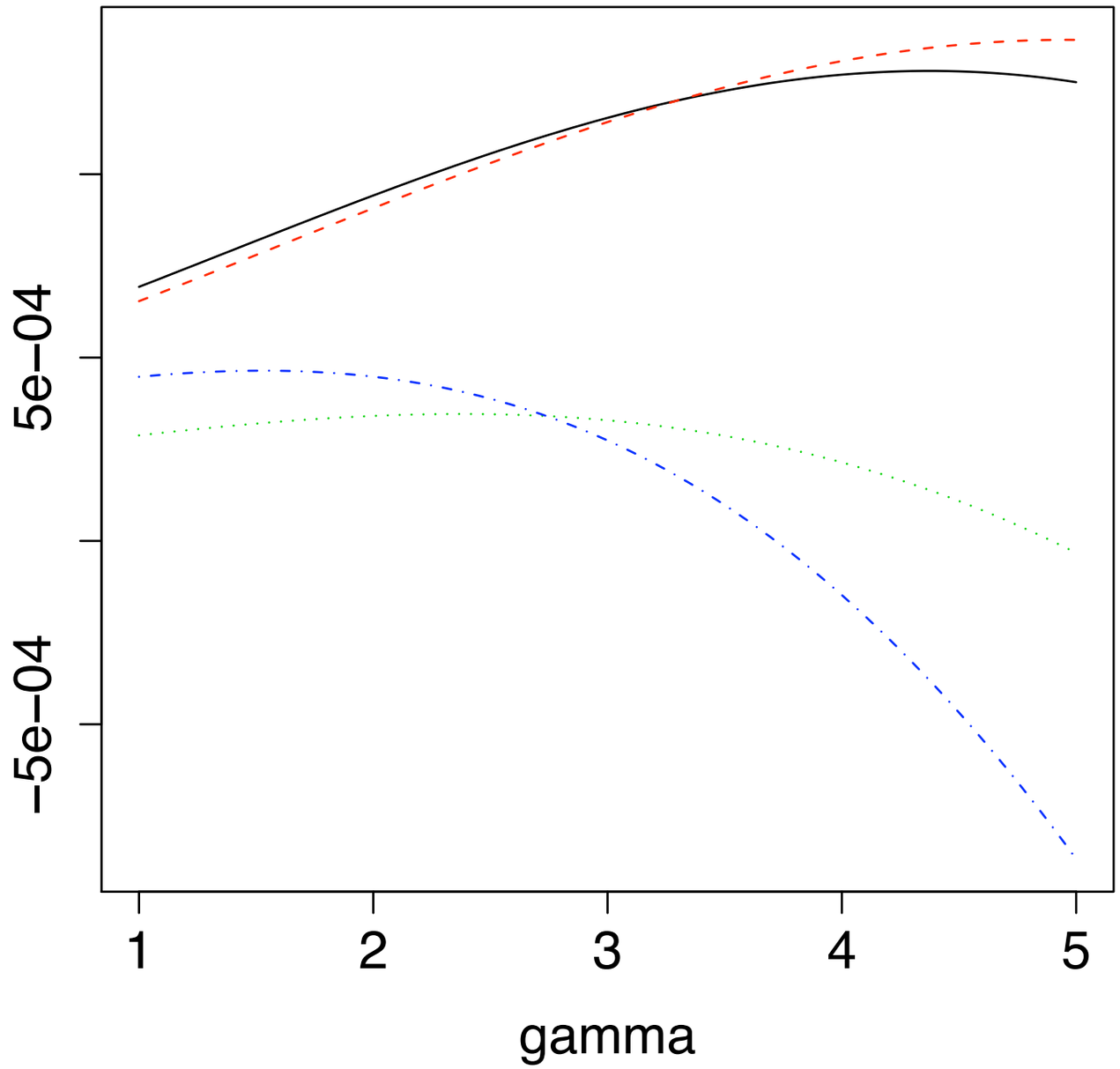
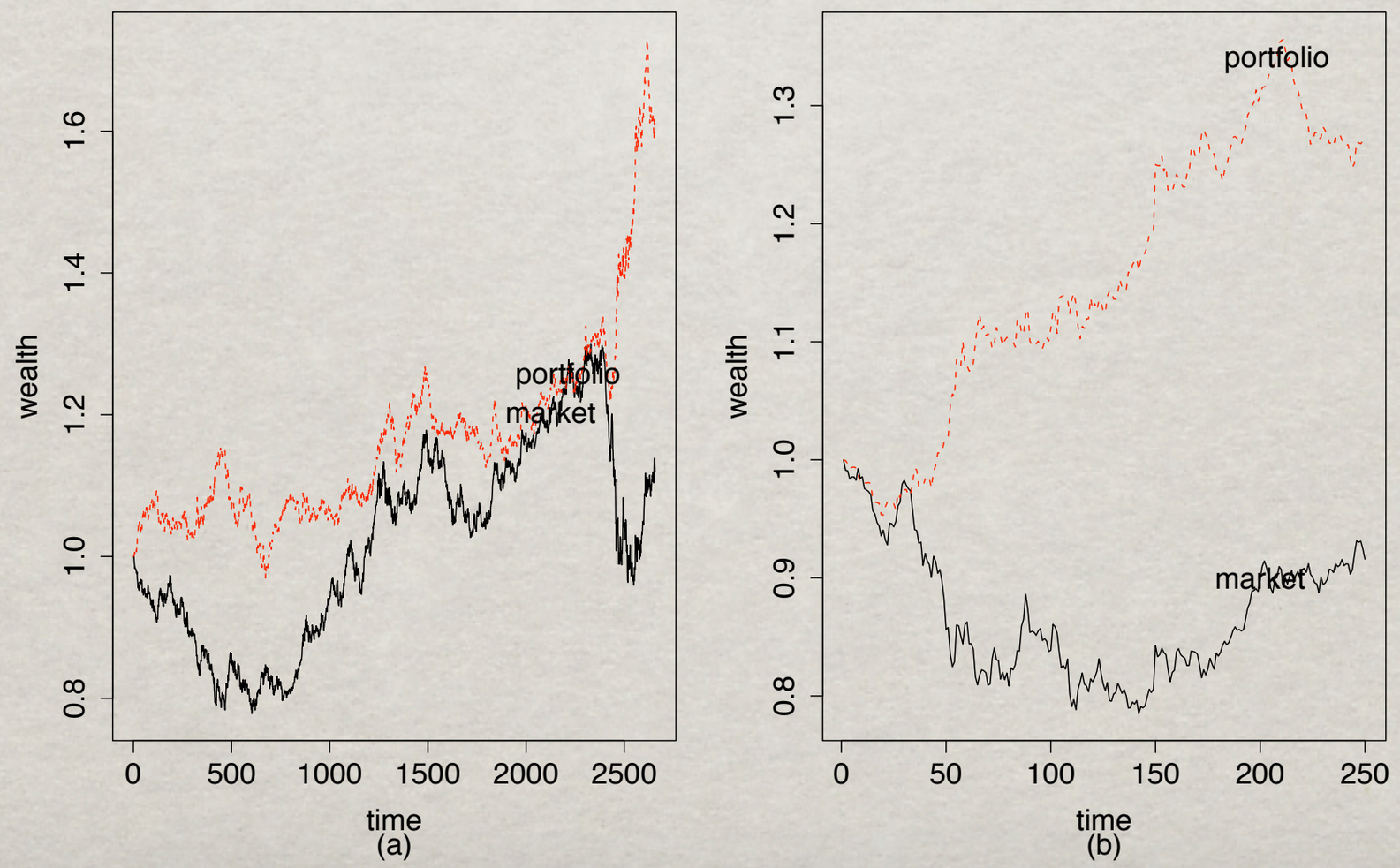


Figure 15: Expected utility curves for the portfolio with $\gamma = 1$ (black), for the portfolio with $\gamma = 10$ (red), for the S&P 500 (green), and for the NASDAQ-100 (blue). The portfolio is the S&P 500–NASDAQ-100 portfolio with shorting allowed. The period extends from 1 October 1985 to 4 May 2009.

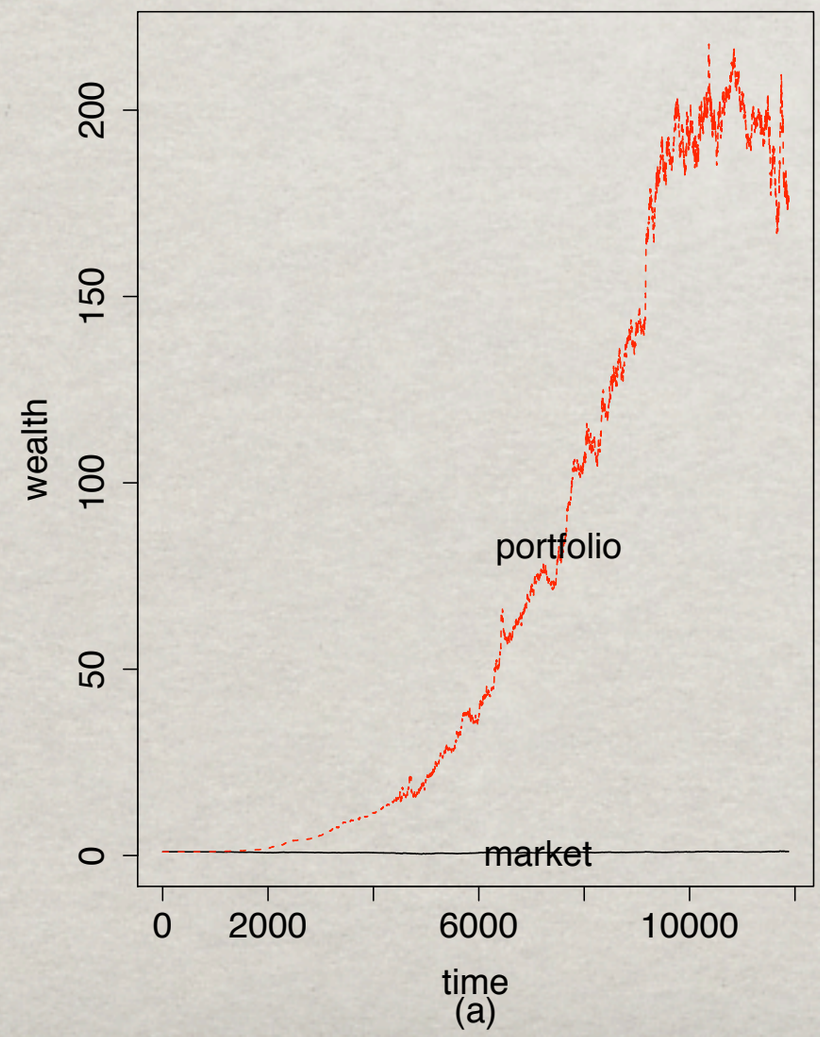
Euro, UK Sterling

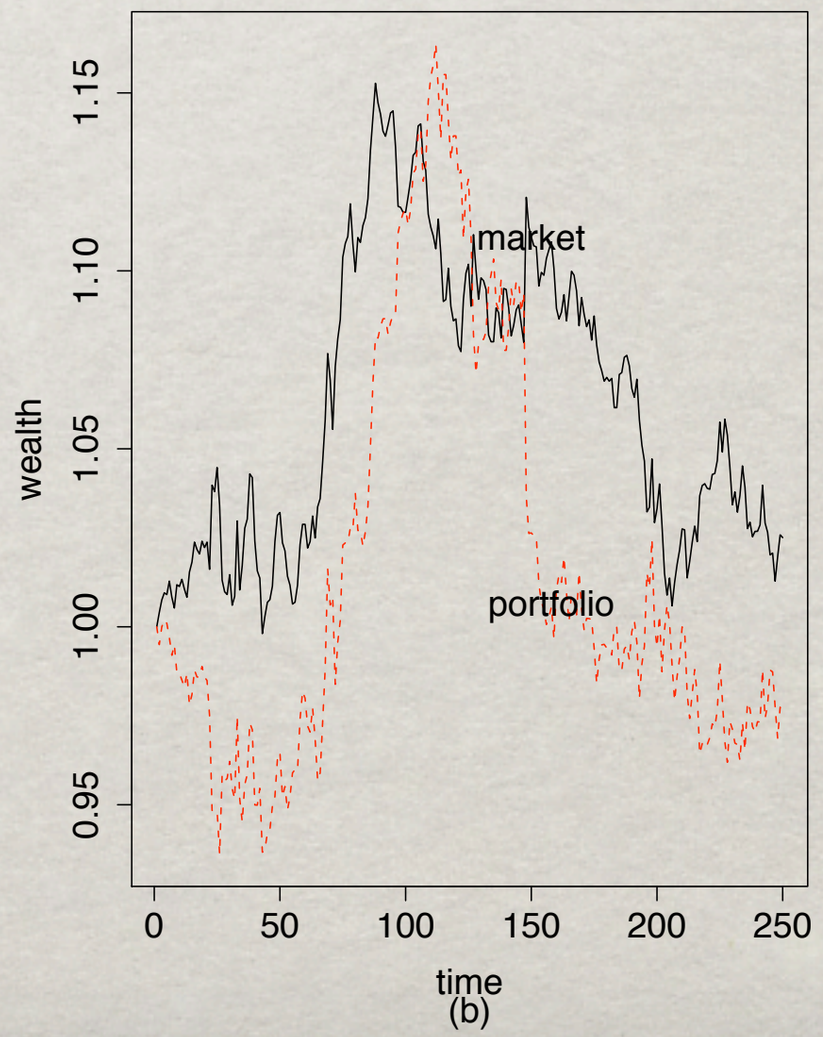
- Shorting allowed, combining strategies
- 1999-04-01 to 2009-08-07
- Mean ret 4.8%, sd 7.4%, Sharpe ratio 0.65



U.S. 10 yr, 5 yr Bond

- Shorting allowed, combining strategies
- Ret 11.4%, sd 9.8%, Sharpe ratio 1.16





Summary

- We can improve market portfolios using kernel regression
- The dependence between indexes can be utilized